1966-05

Extracting analogue signals from noise using a digital computer

Barrett, Neil A.

Monterey, California. U.S. Naval Postgraduate School

EXTRACTING ANALOGUE SIGNALS FROM NOISE USING A DIGITAL COMPUTER

NEIL A. BARRETT

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EXTRACTING ANALOGUE SIGNALS FROM NOISE

USING A DIGITAL COMPUTER

by

Neil A. Barrett
Lieutenant, Royal Canadian Navy
B.S.E.E., Nova Scotia Technical College, 1958

Submitted in partial fulfillment
for the degree of

MASTER OF SCIENCE IN ELECTRONICS ENGINEERING

from the

UNITED STATES NAVAL POSTGRADUATE SCHOOL
May 1966

ABSTRACT

It is frequently convenient in data processing to convert analogue to digital data for computer assimilation. A convenient method of such conversion has been developed and used in the study of correlation detection of audio signals corrupted by noise.

A method to use apriori knowledge of the corrupting noise to increase processing gain has been studied. In the case of detection of a sinusoid in noise, an additional gain over conventional auto-correlation of up to 14.5 db has been achieved.

Finally, a signal source located in an unknown random noise field was detected, classified and located in relative bearing by the cross-correlation of the signals received from two spatially separated sensors.

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1. Introduction.

During the summer of 1965, the author visited the U.S. Naval Electronics Laboratory, San Diego, for six weeks. The experience gained there suggested several problem areas which would profit from further study.

It appeared that there was a need to analyze large quantities of analogue data collected during various experiments. The volume of the data almost precluded "hand" analysis and yet no facilities existed to enable a computer to assimilate the data. This facility was also lacking at the U.S.N. Postgraduate School Monterey.

The data collected during these experiments at San Diego was often corrupted by unavoidable locally generated noise. It was felt that if more could be learned about this self-noise some better method of data extraction might be possible.

One of the most powerful methods of data analysis is the correlation technique which may reveal information masked by corrupting noise. These techniques have been thoroughly investigated over the past 25 years, and a wealth of literature exists on this subject.

Although continuous time integrations and correlations are possible mathematically on known explicit functions, physical data often does not fit a known model. In this case approximations to infinite integrals etc., must be made using either analogue or digital equipment. Analogue equipment tends to be limited in versatility and is expensive, while if a general purpose digital computer is available, it can be made to simulate the physical

situation. Therefore attention was directed toward describing the analogue data by a digital computer. The use of a digital computer to compute various correlation functions is the subject of this report.

2. Statement of the Problem.

The correlation function of two continuous time variables is given by

$$R_{S+N}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} s(t)\, N(t+\tau)\, dt \qquad (1)$$

where s(t), N(t) are defined for $-\infty < t < \infty$

When the functional form of s(t) and N(t) are known it may be possible to evaluate Eq. (1)mathematically. When the form of s(t) and N(t) are known only as analogue signals Eq.(1) may be instrumented by a multiplier and an integrator.

The same effect may be achieved by digital means, subject to errors to be discussed.

The sampling theorem states that a continuous function can be represented in a finite interval by a finite number of samples of the function. The rate of sampling must exceed twice the highest frequency component of the Fourier transform (spectrum) of the function.

By suitably band-limiting a function, samples may be taken at a practical rate. The samples if digitally described can then be processed. Equation (1) implies a memory extending into the infinite past which cannot be achieved physically. As an approximation the "short-time" correlation may be formed with some resulting error.

If the "short-time" correlation is formed digitally, errors arise from both the limited number of samples and by the finite accuracy of the individual samples. If "short-time" is defined to be some period much longer than the period of the lowest frequency in the signal, errors of the first kind above tend to zero as the length of record grows. Measurement error is negligible if the samples are described to four place accuracy.

The "short-time" correlation is formed as

$$\hat{R}_{S^*+N^*}(k\Delta T) = \frac{1}{N-K} \sum_{i=1}^{N-K} s_i^* \; N_{i+K}^* \tag{2}$$

where

ΔT - the sampling interval

s_i^*, N_i^* = sample values of the continuous function taken at intervals of T

$k = o, 1, 2 \ldots\ldots K$

N = the number of samples available.

In the limit as N goes to ∞, and $\Delta T \rightarrow 0$

$$\hat{R}_{S^*+N^*}(k\Delta T) = R_{S+N}(\tau) \tag{3}$$

For finite N

$$\hat{R}_{S^*+N^*}(k\Delta T) \doteq R_{S+N}(\tau) \tag{4}$$

Equation (2) holds only if the data has zero mean. If in Eq. (1) we assume that s(t) is the desired signal, of zero mean and that N(t) is random Gaussian noise and that the observed V(t) = s(t) + N(t), then we may form $R_{VV}(\tau)$. This function may in turn be broken down into components:

$$R_{VV}(\tau) = R_{SS}(\tau) + R_{NN}(\tau) + R_{SN}(\tau) \tag{5}$$

Since the noise and signal are assumed independent the third number vanishes in the limit $T \to \infty$.

We may approximate Eq. (5) as

$$\hat{R}_{S*+N*}(k\Delta T) = \hat{R}_{S*S*}(k\Delta T) + \hat{R}_{N*N*}(k\Delta T) \tag{6}$$

in which we wish to determine $R_{S*S*}(k\Delta T)$.

For very large $k = K_o$ the second member of Eq. (6) will tend toward zero because of independence leaving

$$\hat{R}_{S*+N*}(K_o + j)\Delta T \doteq \hat{R}_{S*S*}(K_o + j)\Delta T \tag{7}$$

$$j = o, 1, 2, \cdots\cdots J$$

This restriction is not desirable since if $R_{N*N*}(k\Delta t)$ were known for all k, then in Eq. (6)

$$\hat{R}_{S*S*}(k\Delta T) = \hat{R}_{S*+N*}(k\Delta T) - \hat{R}_{N*N*}(k\Delta T) \tag{8}$$

However $R_{N*N*}(k\Delta T)$ is never known exactly. It can only be known, since it is a random variable, in an average sense.

If we assume that the normalized correlation function $\bar{R}_{NN}(k\Delta T)$ is known apriori

$$\bar{R}_{NN}(k\Delta T) = \frac{1}{\alpha} R_{N*N*}(k\Delta T) \tag{9}$$

where $\bar{R}_{NN}(k\Delta T)$ is formed in the absence of signal

α is to be determined where $0 \leq \alpha \leq 1$

then in Eq. (8) we may write

$$\hat{R}_{S*S*}(k\Delta T) \doteq \hat{R}_{S*+N*}(k\Delta T) - \alpha \bar{R}_{NN}(k\Delta T)$$

Methods of determining α have been studied for the very limited case

of a single sinusoid signal masked by noise.

To gain some experience with the technique of cross-correlation, a low level signal was placed in a noise field. The output of two spatially separated sensors was processed to detect and locate the signal. The results are presented in section 5.

3. Conversion of Analogue Signals to Digital Form for Computer Analysis.

Many analyses of real data may be performed on a digital computer by first manually sampling the data a sufficient number of times, and entering the sampled data into the computer.

When the number of data samples desired exceeds a few thousand, some automatic means of data conversion must be used.

The method should preferably be "off-line," that is, use auxiliary equipment, for efficient employment of the main high speed computer. The Computer Laboratory of the Electrical Engineering Department was equipped with the necessary items of hardware and these facilities were used to sample the analog data and form the digital record.

The three main items used were a 12 bit analogue to digital convertor a Control Data Corporation (C. D. C.) Model 160 Computer having a capacity of 4096, 12 bit words, and a C. D. C. Model 163 magnetic tape unit.

As now constituted the analogue data is consecutively sampled by the analogue to digital converter working under control of the CDC 160 computer.

When 4000 samples have been taken, the computer writes the samples on magnetic tape in a format that is recognized by the main CDC 1604 com-

puter. After a delay of 0.4 second another 4000 samples may be collected, and so on. Each group of 4000 samples when written on tape is accompanied by an identifying word so that any block of 4000 samples may later be located.

The maximum sampling rate is 5 Kcs. implying that all signals should be band-limited to less than 2500 cps. The sampling rate may be varied at will by manual program entries, as can other convenient parameters.

The system is capable of digitizing only one source at a time. To allow cross-correlation of two or more sources, a clock pulse input is used to initiate a round of sampling.

At present a simple AND gate is used to sense the presence of a clock pulse. A more permanent arrangement should make provision for the detection of a simple clock code, to eliminate the occasional false starts due to noise pulses, which have occured under present arrangements.

Once the data has been recorded digitally on tape, the data may be read into the main computer using standard programming techniques. The subroutines to accomplish this are available as Subroutine Data from the Computer Facility.

The data when read into the computer is delivered in integer format, i.e., without a decimal. To convert it to units of volts each data point is then divided by 409.6, a factor inherent in the particular A/D converter used. The diagram below shows the essential steps.

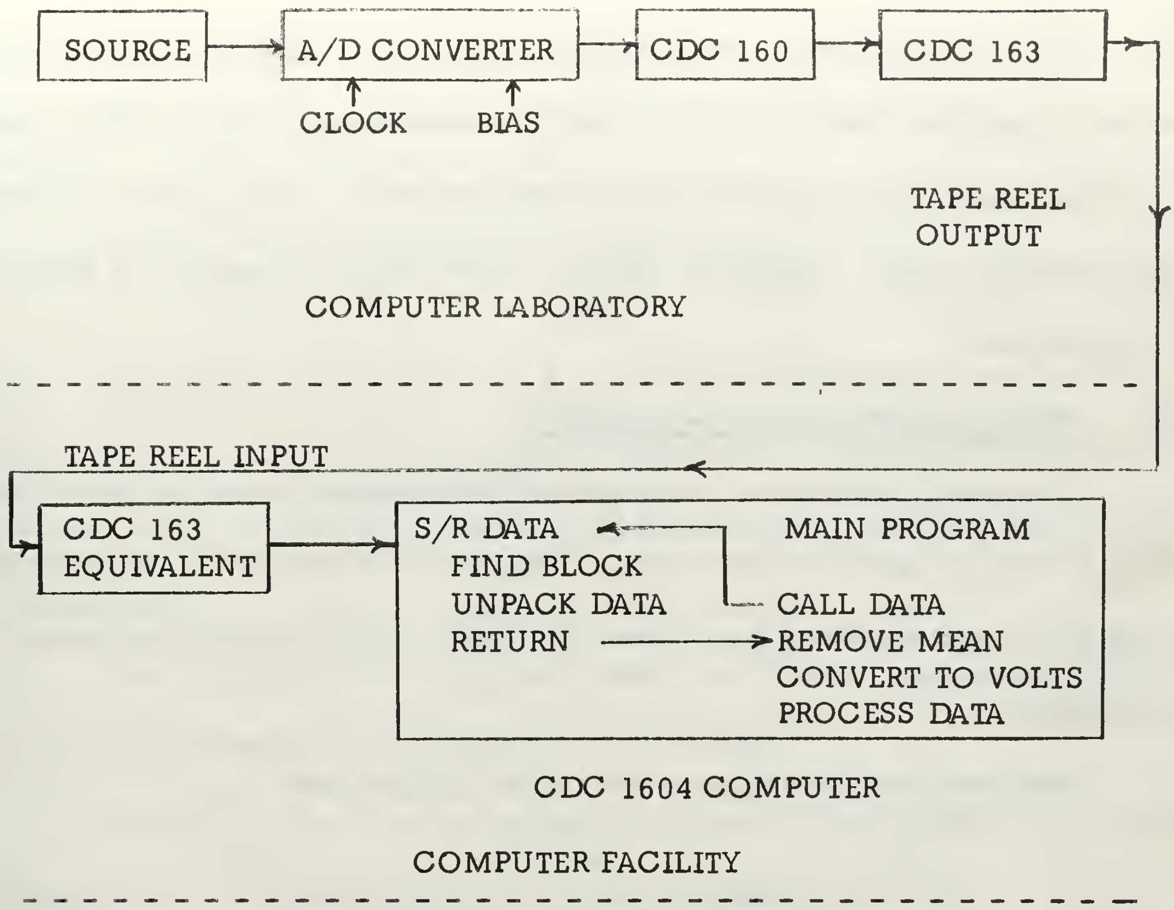

Figure 1. Flow Diagram of Digitizing Process.

The system is presently adequate to describe many physical processes. Certain signals, such as speech, are beyond the capabilities of the system. However modifications to the Computer Laboratory hardware to improve performance were not attempted because of pending contractual arrangements to completely replace the hardware with more sophisticated equipment. The new equipment will enable two records to be digitized simultaneously. The memory capacity will be increased by nearly a factor of four, and the sampling rate will be approximately 20 Kcs. This equipment will be available for use in mid 1967.

The author has recorded the mixed output of a signal and a noise generator to provide a vehicle for the study of correlation. Several other students [6], [7] have used the system for their own particular needs with no difficulty experienced to date. Additional details on the digiting process are presented in Appendix A.

4. Noise Removal in Auto-Correlation.

Correlation techniques have gained considerable favour in recent years as a method of signal enhancement. Theoretical derivations of the processing gain to be expected have been stated by Lee[1], for both auto and cross-correlation.

Auto and cross-correlation are defined as follows:

$$R_a(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} V(t) . \; V(t + \tau) \, dt \qquad (1)$$

where $V(t) = s(t) + N(t)$

$s(t)$ = the signal to be recovered

$N(t)$ = noise, here assumed Gaussian

$$R_C(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} V(t) . \; s^1(t + \tau) \, dt \qquad (2)$$

where $s^1(t)$ is chosen apriori to have the form of $s(t)$ and is locally generated.

Approximations to Eq. (1) and (2) may be generated using sampled data in which case.

$$\hat{R}_a(\tau) = \frac{1}{N} \sum_{i=1}^{N} \hat{V}_1(i) . \hat{V}_1(i + \tau) \doteq R_a(\tau) \qquad (3)$$

$$\hat{R}_C(\tau) = \frac{1}{N} \sum_{i=1}^{N} \hat{V}_1(i) - \hat{V}_2(i+\tau) \simeq R_C(\tau) \tag{4}$$

where $\hat{V}_i = V(t)$ for $t = i\Delta t$

It may be shown that Eq. (3) and (4) are of the form

$$\hat{R}_{s+N}(\tau) = \hat{R}_{ss}(\tau) + \hat{R}_{NN}(\tau) + \hat{R}_{sN}(\tau) + \hat{R}_{Ns}(\tau) \tag{5}$$

Of the four terms in Eq. (5) it is desired to determine only the first, $\hat{R}_{ss}(\tau)$ which represents the signal contribution. The remaining terms are undersirable and success is measured by the extent to which they can be eliminated.

The cross-terms $\hat{R}_{sN}$, $\hat{R}_{Ns}$ will vanish as $N \to \infty$, because of the assumed independence of the signal and the noise.

Since the correlation is to be done using a finite number N of samples, these terms will become small, but may be significant.

Let us assume that a signal is of the form

$s(t) = \sqrt{2}\, E \cos wt$

$N(t) =$ Gaussian noise of zero mean and variance σ_N^2

The variance of the output of Eq. (3) and (4) has been evaluated by Lee [1], who clearly shows the superiority of cross-correlation over auto correlation.

In the case of auto-correlation

$$\sigma_S^2 = \frac{E^4}{2} + 2E^2\sigma_N^2 + \sigma_N^4 \tag{6}$$

while for cross-correlation

$$\sigma_S^2 = \frac{E^4}{2} + E^2\sigma_N^2 \tag{7}$$

This improvement is achieved only when the form of the desired signal is known apriori. In many cases of interest particularly in the passive sonar situation, the desired signal form is not known apriori, and consequently auto-correlation must be used.

In addition to the noise input supplied by the environment, local disturbances may cause noise effects to appear as part of the correlator output. If these two sources of noise, taken together, are known apriori, and can be considered to remain stationary over the interval under consideration, then perhaps some means can be found to eliminate their corrupting effect on the output of the auto-correlator.

This problem has been studied for many years. In an early work Lee [4] states

> "...the correlation of the noise, once measured can be compensated. (for)"

The details of the implementation of this were not specified.

In a much more recent work Van Trees [3] considers an optimum receiver in which he considers that the noise is known apriori as a normalized correlation $\gamma(\tau)$. After correlation of the received data some constant K times $\gamma(\tau)$ is substracted form the correlator output. Determination of K rests upon knowledge of signal strength and noise strength.

Again it is not clear how to instrument the calculation of K.

We will assume that the sample auto-correlation of the signal, $\hat{R}_{S+N}(\tau)$ has been formed in the presence of corrupting noise. We shall also assume that the statistics of the corrupting noise are known apriori and are represented by the normalized correlation function $\bar{R}_{NN}(\tau)$ formed in the absence of signal.

The object of the processing is to determine a linear combination of $\hat{R}_{S+N}(\tau)$ and $\bar{R}_{NN}(\tau)$ which will enhance the signal and tend to remove the noise. No claims are made for the optimality of the method presented here, other than it does improve the signal to noise ratio of the output.

In the following we will be dealing with sampled data functions, not continuous functions of time. To simplify notation however the symbol τ will be used to denote a sample data function.

The power spectral density P(w), and the auto-correlation functions are transform pairs, that is

$$R(\tau) = \int_{-\infty}^{\infty} P(w)\, e^{jw\tau}\, dw \tag{8}$$

$$P(w) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R(\tau)\, e^{-jw\tau}\, d\tau \tag{9}$$

Given one member of the pair, the other may be found. Therefore the processor could attempt to operate on either the power spectral density or the correlation function.

From Eq. (5) we get (neglecting cross terms),

$$\hat{R}_{SS}(\tau) = \hat{R}_{S+N}(\tau) - \hat{R}_{NN}(\tau) \qquad (10)$$

or equivalently by Eq. (9)

$$P_{SS}(w) = P_{S+N}(w) - P_{NN}(w) \qquad (11)$$

We have assumed that as apriori information estimates of the normalized power spectral density function $P^*_{NN}(w)$ and normalized auto-correlation function $R^*_{NN}(\tau)$, formed in the absence of signal, are available. At any given instant there is no assurance that the apriori known noise estimate represents the true noise in other than an expected value sense.

Then in Eq. (10) and (11) we may write

$$R_{SS}(\tau) = R_{S+N}(\tau) - \alpha\, R^*_{NN}(\tau) \qquad (12)$$

$$P_{SS}(w) = P_{S+N}(w) - \alpha\, P^*_{NN}(w) \qquad (13)$$

where $R^*_{NN}(\tau)$ and $P^*_{NN}(w)$ are known functions

$0 \leq \alpha \leq 1$, α a constant to be determined.

Determination of the α in Eq. (12) and (13) becomes the central problem to be considered.

The following methods of calculating α hold only for the case of sinusoidal signals in noise. Further study is necessary before any general method can be stated.

Since the correlation function was produced from the data directly and the power spectral density indirectly from the data via the correlation, it was thought advisable to operate first on Eq. (12) rather than Eq. (13).

In the case of a sinusoidal signal, it was thought that finding an

$$\alpha = \alpha_C, \text{ such that } R_{SS}(0) \gg \underset{\tau \neq 0}{\text{Max}}\, R_{SS}(\tau) \qquad (14)$$

would be advisable.

However Eq. (14) is only a necessary not a sufficient condition for the formation of a valid, physically realizable auto-correlation function.

Thus an instrumentation of Eq. (12) to satisfy Eq. (14) may result in a correlation function which is the output of a physically unrealizable linear system. In the time domain there seem to be no simple tests for realizability.

Several signals were analyzed using Eq. (12) and (14) at input signal to noise ratios of 0 and -10 db. No trouble was experienced with the 0 db case; the signal was almost completely recovered and the noise suppressed. The -10 db input signal after similar processing was transformed into the frequency domain. The signal was clearly evidenced by a large spike in the power spectral density at the signal frequency. The amplitude of the signal spike after processing showed nearly 6 db greater discrimination between signal and noise peaks than it did before processing.

The above gain would have been encouraging had not the power spectral density showed some negative contributions.

Negative contributions in a power spectral density, are evidence of, and in fact are a sufficient condition for, physical unrealizability.

Because of this difficulty no further efforts were made to suitably determine α_c on the basis of the correlation functions. Efforts were then directed to a determination via Eq. (13).

If the signal is a pure sinusoid with no noise present, the power spectral density shows a delta-function $\delta(w)$ at $w = w_S$, and is identically zero for all other values of w. The perfect processor would give this output even in the presence of noise.

There are two conditions one may then place on Eq. (13). These are

(a) There must be no negative contributions $0 < w \leq w_{Max}$

(b) At least one point must be zero.

This allows an α_c to be determined directly in the frequency domain. Condition (a) above ensures realizability, while (b) ensures that for any $\alpha > \alpha_c$, condition (a) is violated. On the other hand choice of an $\alpha < \alpha_c$ results in the removal of less noise than one might achieve otherwise, but at least ensures physical realizability.

A trial and error search routine was carried out to find an α which satisfies conditions (a) and (b) above. The search routine is halted when an α is found which results in at least one point of Eq. (13) having a value ϵ, $0 \leq \epsilon \leq .00001$, and no negative points. The time to complete a search is in the order of .5 seconds, which is small compared to the time taken to form the functions originally (70 seconds).

Having formed $P_{SS}(w)$ in Eq. (13) the results were renormalized to make the area under the curve unity, and then plotted.

The α_c determined by the search was then used in Eq. (12) to form $R_{SS}(\tau)$ and the result plotted. No claims for validity are made for this step except that having formed $R_{SS}(\tau)$ in this manner its transform had no negative

components.

The noise removal proceedure based on an α determined from the power spectral density resulted in an additional 14.5 db processing gain. This figure was obtained by considering the ratio of energy under the 50 cps. comb-width to the total energy under all other comb-widths, both before and after the noise removal. Before noise removal the calculated input signal-to-noise ratio was -8.84 db. After noise removal the equivalent calculation yielded + 5.62 db.

Another measure of effectiveness should be mentioned. In many cases the ability to distinguish a signal from noise in the spectral density is of interest. In the power spectrum the ratio of the amplitude due to signal to that due to a noise "spike", may be expressed in db. In the -8.84 db (nominally -10 db) case, the ratio of signal to highest noise peak was 5.62 db. before noise removal, and 9.05 db. after noise removal, or an improvement of 3.4 db.

Details of the processing are included as Appendix B.

5. Location of a Source by Cross-Correlation.

The bearing of a signal with respect to an observer may be determined by measuring the difference in arrival time of the signal at two or more separated observation stations.

As an experiment a 200 cps. sinusoidal signal generator was set in one corner of a cafeteria. Two microphones spaced a distance d units apart were located symmetrically with respect to the center line of the room, at the far wall of the cafeteria. Interfering noise was supplied by the noon-day

clientele.

The maximum level of signal was limited by the intolerance of the diners. This was determined to be 38 milli-watts measured at the loudspeaker voice-coil. At this level the signal was inaudible at the microphones.

A stereo recording was made of the signal plus noise, and after slowing the tape by a factor of four, selected portions of data were digitized using techniques detailed in Appendix A. The resulting digital data was cross-correlated in the CDC 1604 computer and the correlation function plotted.

The geometry of the problem is shown below.

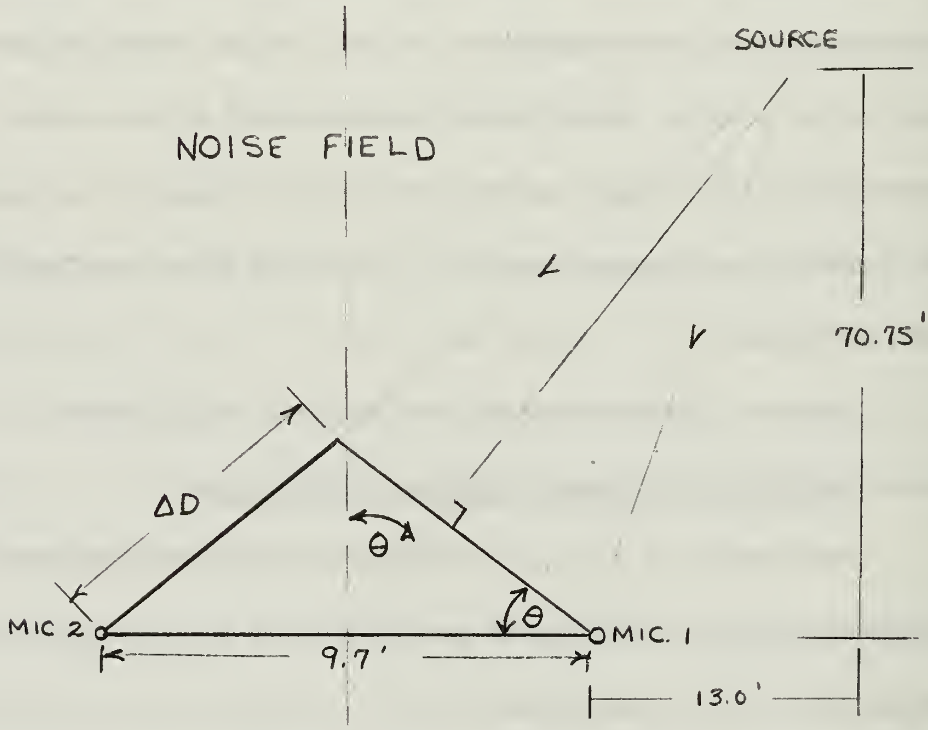

Figure 2. Geometry of Lunch-room Problem.

Assuming plane wave propagation, and in the absence of noise, we may associate with Δ D in Figure 1, an equivalent amount of time; that is,

$\Delta T = \Delta D/C$ where C is the velocity of sound in air.

If the two sensors are cross-correlated a maximum in the function will occur whenever the time variable of correlation is equal to a multiple of the period of the signal.

The signal was located successfully to an accuracy of $\pm 0.17^{\circ}$. The frequency of the observed correlation function was nearly 150 cps., or three times the expected value of 50 cps. This would indicate that detection was accomplished on the third harmonic of the source oscillator.

A study of the noise characteristics supplied by the diners was not made.

Provided that accurate time synchronization between the two data sources to be correlated can be maintained during the digitizing process, correlation will provide both bearing and frequency information.

Details of the problem are presented as Appendix C.

6. Conclusion.

A convenient method of digitizing analogue data has been developed and this digital data used to detect signals masked by noise.

The digitizing system is now operational.

Many interesting areas of research remain to be pursued in the field of recovery of noisy signals. In particular additional effort is needed in the area of removal of apriori known noise effects.

The facilities to perform the necessary calculation on a digital computer are now available at the U.S. Naval Postgraduate School.

ACKNOWLEGEMENT

The author extends his thanks to Dr. G. Myers of the Electrical Engineering Department for his thoughtful guidance and help.

Dr. L. V. Schmidt of the Aeronautical Engineering Department has been vitally interested in the development of the digitizing process and has lent his time and his energy to help accomplish this goal.

Mr. E. Janzen of the Programming Staff of the Computer Facility has been extemely helpful with details of programming and has been of great assistance.

Finally, I would like to thank the Staff of the U. S. Naval Electronics Laboratory, San Diego, in particular Mr. W. Littrell and his staff, for the courtesies extended to me during my visit of June 1965. The experience gained there largely motivated this work.

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APPENDIX A

ANALOGUE TO DIGITAL DATA CONVERSION

1. INTRODUCTION.

The process of translating real, analogue data into computer digitized form has been accomplished in two separate stages.

In the first stage, the analogue data on magnetic tape is sampled at discrete points of time. These samples are digitized and written in bit form on magnetic tape.

In the second stage, the digital magnetic tape is loaded on the CDC 1604 peripheral tape units and then read into the computer.

Earlier work at USNPG School in computer analysis of analogue data utilized the "on line" approach, in which the data was fed to the main computer directly from the digitizing computer. This mode of operation is now impractical because of high computer facility utilization.

The method detailed here results in data being reduced to a form suitable for submission as a standard program input.

2. DIGITIZING THE DATA.

Basically the method takes samples until the Core storage of the digitizing computer is full, and then writes this information on magnetic tape in digital form.

The digitizing computer is a CDC 160 located in the Computer Laborotory of the Electrical Engineering Department.

This computer has a 4096, 12 bit memory length, and has the ability to select various external equipments through the proper external function

codes. The lower 4000 cells of the computer were reserved for data storage and the top 96 cells for programming. Thus the core storage available sets one quite restrictive limitation on the system. In practice however, 4000 samples were adequate to describe many common processes.

The block diagram of the system is shown in Figure 1. The 160 selects either the A/D converter, for data input, or the 163 tape unit for data output. When the A/D converter is selected (EXF 14xx) and the INA (INput to A) instruction is executed, the input analogue signal, is sampled, and a 12 bit number representing amplitude is sent to the 160 and stored. Repetition of the process in quick succession allows blocks of up to 4000 samples to be stored. The maximum sampling rate achievable is 5000 samples/second.

To control exactly the time at which samples are taken, the execution of the INA instruction is prevented until two events have occurred; first, an enabling switch must be closed to ground (EXF 2410), and second, a pulse (see Figure 2) must be present at the base of the AND Gate. When these conditions have been satisfied the flip-flop goes to the "1" state placing -3 volts on the INA line and thus executing the INA instruction.

Further samples may now be taken as the flip-flop will stay in the "1" state until EXF 2400 is called, resetting FFI to the "0" state. The complete process is illustrated in Figure 3.

The usual mode of operation is to have each block initiated by a pulse, with the intersample delay under computer control. Under certain circumstances, it may be desired to have only the runs initiated by a pulse, with both

the block count and the sample count under computer control.

The inter-block timing in this mode is not accurate enough for cross-correlation. The basic program shown in Table 1 may be modified using Table 2 to give various modes of operation.

Operating Instructions-Digitize Eq. (3).

Features of Digitize Eq. (3).

1. Manual Entries

   a. 0066 - intersample delay. See Figure 6

   b. 0067 - any identifier you wish

   c. 0070 - initial run number

   d. 0071 - number of runs desired

   e. 0072 - number of samples/block $\leq 7640_8 = (4000)_{10}$

   f. 0073 - number of blocks/run

Note that (d) x (f) $\leq 760_8$

2. Main Points

   a. A block of n, $n \leq 4000_{10}$ samples is taken each time a clock pulse is present and EXF 2410 is made.

   b. The inter-sample spacing interval is program controlled and is variable from approx. 190 μsec. minimum to about 50 msec. maximum.

   c. The first four (4) words written on the 163, of each block, are identifying words in the following order:

   (1). Run number.

(2). Spare (may be preset to anything)

(3). Block number

(4). Number blocks/run

d. The identifying information when digitizing data must be recorded, as 1604 subroutine DATA uses this information to locate blocks of data on digital tape.

e. At the end of each run a check is made to ensure that the storage capacity of a full roll of 163 tape is not being exceeded. If it is, an End of File is written and the program halts with the total number of blocks written in the A register.

f. Program may be stopped at any point to allow cueing data tape, provided that it is not cleared. Progress resumes when 160 placed in run condition.

g. Analogue input is at A/D Channel 1. A voltage reference is established by an external source at about -5 volts. Correct operation is assured if the proper test pattern is displayed. See instructions for Program Test 160.

h. A voltage of V volts at A/D Channel 1 is converted to a digital reading by the A/D converter, equal to -409.6 x V

3. <u>Operating Instructions</u>

a. Load from cell 0000

b. Run from cell 0000

c. When digitizing is complete program stops at 0124 with current run number in A register.

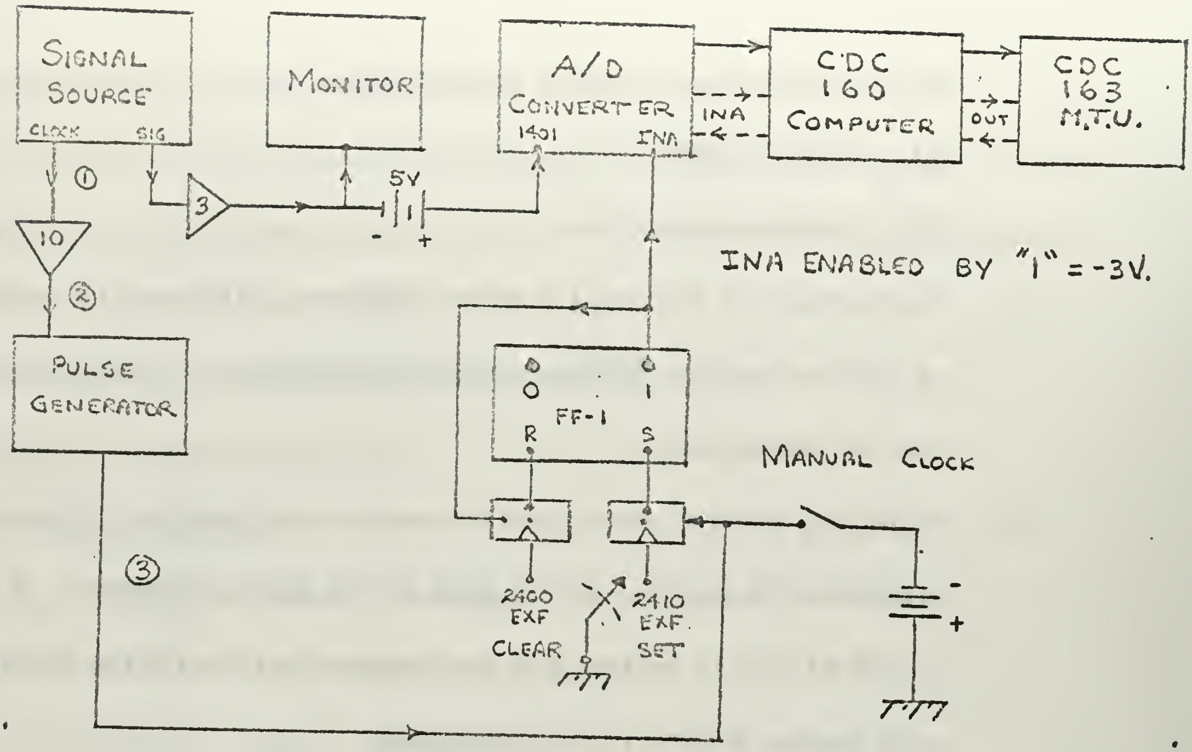


FIGURE 1 - BLOCK DIAGRAM OF DIGITIZING HARDWARE.

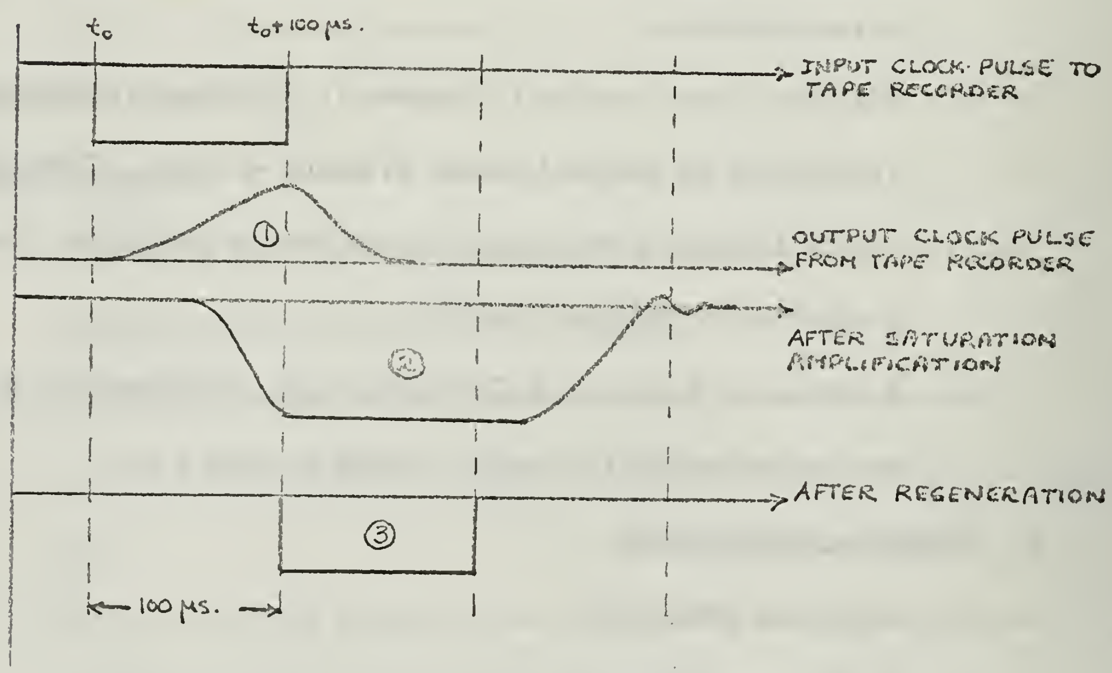


FIGURE 2 - CLOCK PULSE DEVELOPEMENT.

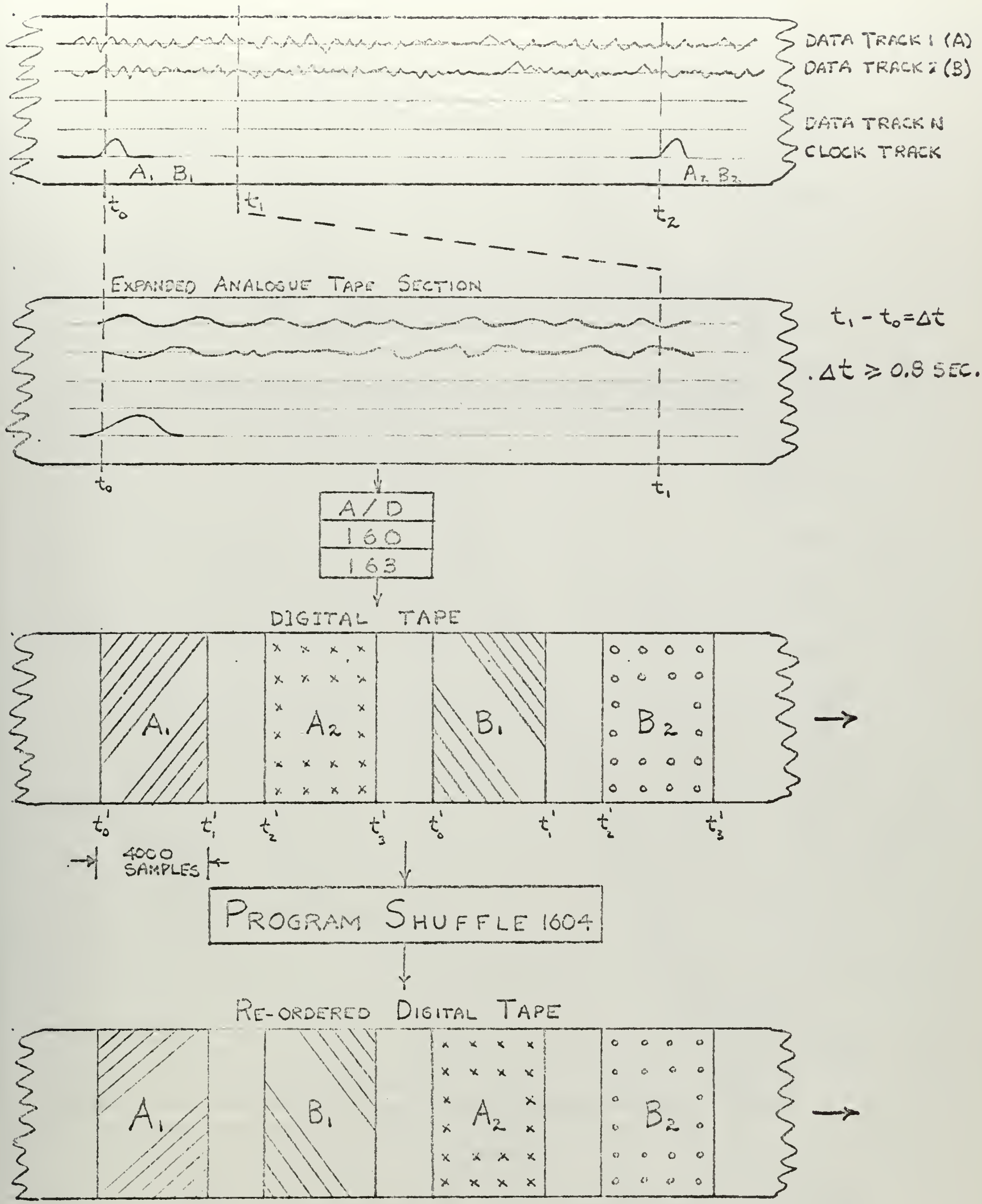


FIGURE 3 - THE DIGITIZING PROCESS

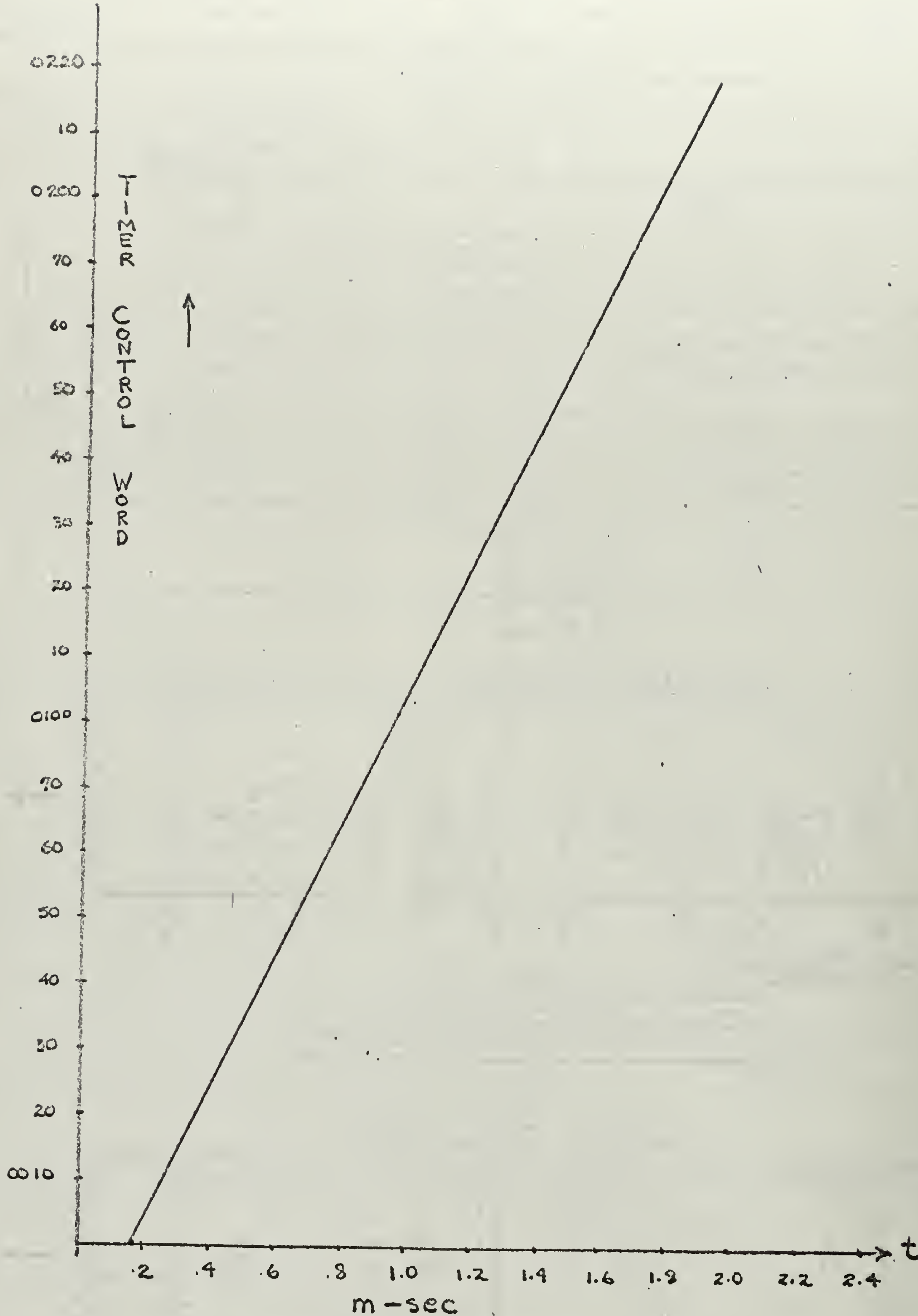
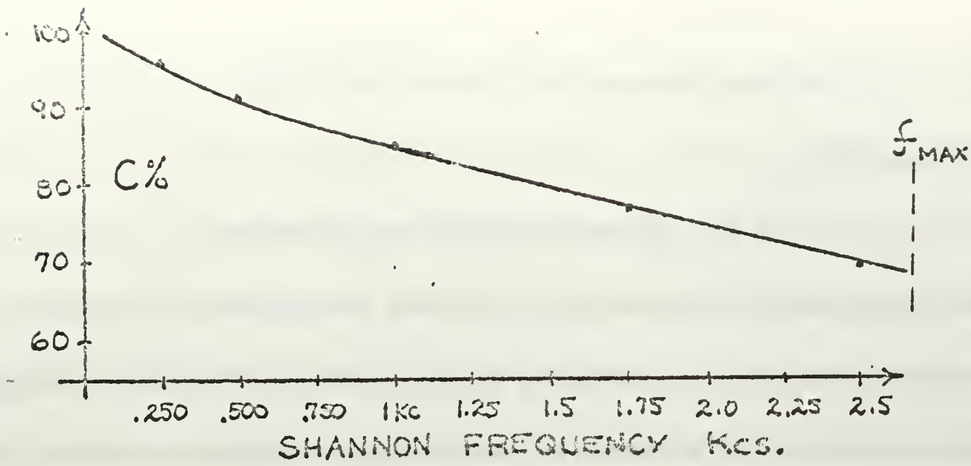
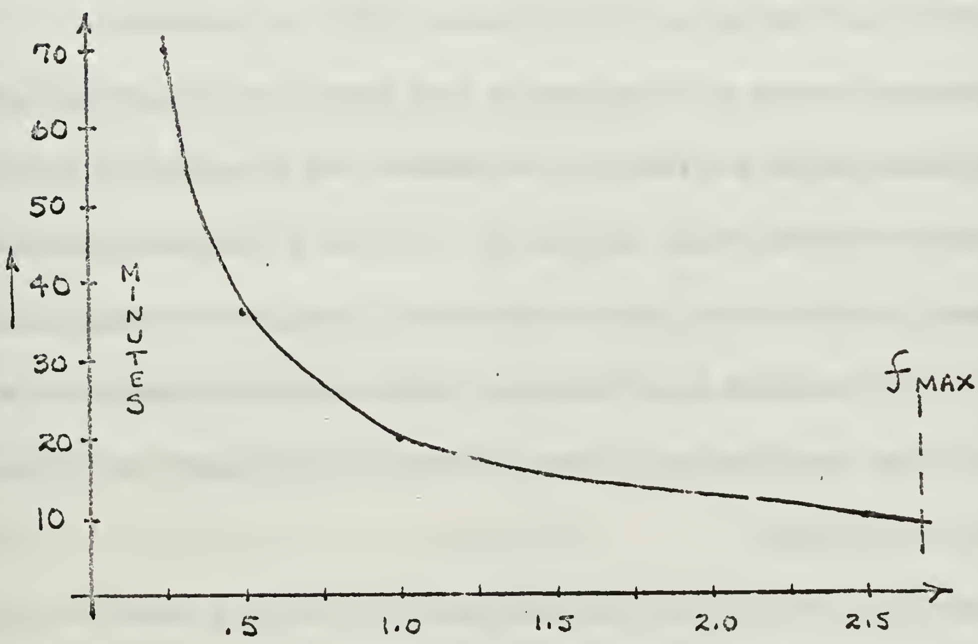


FIGURE 6: TIMER CONTROL WORD VS. SAMPLING INTERVAL



$$C = \frac{\text{INTER-SAMPLE DELAY} * \text{NUMBER SAMPLES/BLOCK}}{\text{INTER-SAMPLE DELAY} * \text{NUMBER SAMPLES/BLOCK} + .35 \text{ SEC.}} \times 100\,\%$$

FIGURE 4



163 MAGNETIC TAPE CAPACITY VS. FREQUENCY

FIGURE 5

d. Clear and run from cell 0127 to write E.O.F.

4. Modification

a. See Table II for various possible modifications.

The limitations of the sampling process are graphed in Figures 4 and 5. Figure 4 shows the effect of sampling speed on the percentage of the data that may be digitized. The speed of the data gathering is a variable under program control, while the speed of output is not. Thus at slower sampling rates, the sampling approaches continuous sampling, and the amount of the real time data which may be represented on tape, increases, as seen in Figure 5.

Figure 6 shows the relationship between the number entered as inter-sample delay timing word, and the resultant inter-sample delay.

The development of a clock pulse from initial recording to output at the flip-flop is shown in Figure 2. Experiment has shown that a total delay of 100 micro-seconds exists between t_o, the time of initiating the pulse on record and t_i, the time the pulse is delivered. This period remains stationary over a large range of level settings, pulse widths and amplifier gain settings. Thus the clock pulse may be used to synchronize two pieces of data, as in cross-correlation.

Experience has shown that since only the first of a series of adjacent clock pulses in used to initiate sampling, the recording of a train of pulses, rather than a single pulse is a great aid in cueing the analogue tape. The length of the train must be less than the length of time taken to sample one block.

In order to set the CDC 160 up properly for digitizing, a short program, TEST 160, has been devised (see Table III). In this program, the first cells -0000 to 0017, contain a program to accurately set the bias voltage. With no signal in and the bias at -5.0 volts, the value of the conversion should be 0000 or 7777. By monitoring D/A Channel 1 with a "scope", deviations from the zero condition may be detected and adjusted using the voltage control on the bias source. Small errors in zeroing may be expected, due to line drift, etc. The effect of this may be eliminated by removing the mean of the data in the 1604.

The cells from 0020 to 0045 contain a program which has identical timing to that of DIGITIZE. By observing the input request pulses on the patch box, between the CDC 160 and the A/D converter, one may adjust the value of the timing control to give the desired inter-sample delay. Changing the control word by one number changes the sampling interval by 12.8 microseconds.

In order to distinctly label each block of data written on digital tape, an identifying series of numbers is first written, followed by the data points. Since it takes four CDC 160 words to equal one 1604 word, it was convenient to specify four quantities in the identifier.

1. Current run number-indexes once a run
2. Spare-may be sent to anything
3. Current block number in the run
4. The number of blocks per run

For example, at the 5th run, using two blocks per run, and the "spare" set at 4321, the identifiers would appear in the 1604 as:

```
0005 4321 0001 0002
0005 4321 0002 0002
```

Precise records must be kept of the identifiers when digitizing data, as data location on tape is by means of these identifiers.

3. DATA RECALL

The data in digital form may be recalled by calling Subroutine DATA. S/R DATA in turn utilizes two subroutines, FINDIT, which locates the data on tape, and UNPACK, a machine language routine which unpacks the samples. The result is to create the original CDC 160 sample list, word for word in the 1604.

The unpacked data is presented to the main program in integer format and appears in either KDATA (M, 1) or KDATA (M, 2) as specified by KLIST.

Integer format output was chosen for the sake of generality as this is the format of the raw unpacked data.

Normal use is to change the data to floating point format, remove the mean, and divide all readings by a factor of -409.6. The letter factor converts all readings to volts.

When reading the tape on the CDC 1604, it is convenient to have the blocks arranged sequentially on tape to prevent tape rewinding and scanning. However, when digitizing data for cross-correlation, it is convenient to digitize all the right channel information, and then the left.

Since it was desired to cross-correlate the left with the right channel,

some reordering of data on tape was necessary. Program SHUFFLE does this, reordering the data so that the output appears as right samples, left samples, right samples, etc., in consecutive order.

Once the data has been written on tape in the order that it will be recalled, the average time taken to recall one block, unpack it, remove the mean, and convert it to floating point form is less than 2.75 seconds.

4. TESTING THE SYSTEM

In order to test the capabilities of the digitizing system, a test tape was prepared with two blocks per run, and each block of 4000 samples on each of six known signals.

Run	Signal	Identifiers
1	8.0v. peak-peak, 250 cps square wave	0001 0000 0001 0002 0001 0000 0002 0002
2	5.0 volt, 250 cps Triangular wave	0002 0000 0001 0002 0002 0000 0002 0002
3	6 volt, 250 cps sine wave	0003 0000 0001 0002 0003 0000 0002 0002
4	-8 volts DC. (-3 below reference)	0004 0000 0001 0002 0004 0000 0002 0002
5	20 cps band-limited white noise, 7.07 volts rms.	0005 0000 0001 0002 0005 0000 0002 0002
6	White noise 7.07 volts rms.	0006 0000 0001 0002 0005 0000 0002 0002

The data was analyzed by Program TEST and the resulting analysis is tabulated in Table IV. The data was recorded on Reserve Tape 18.

Table IV. Summary of Test Results.

Run	Signal	Type	MEAN/EXP MEAN	σ^2/EXPσ^2	fc/EXP
1	Square Wave	250 cps	0/-.132	16/16.365	- - -
2	Triangular	250 cps	0/+.086	2.0/2.09	- - -
3	Sine Wave	250 cps	0/+.027	4.5/4.56	- - -
4	-8 Volts	D.C.	-3/-3.025	0.0/4.74x10^{-6}	- - -
5	20 cps White Noise	Bandlimited	0/+.147	-/2.10	20/29.5 cps
6	White Noise		0/-.094	-/2.2	200/199 cps

Mean Recovery Time Per Block $\leq$ 2.75 Secs.
Digitizing Noise is 63 db Below a 3 Volt Signal

5. ERROR ANALYSIS

In the analysis of the data it was desired to determine an estimate of σ^2, the data second moment, and to determine a confidence interval on the estimator.

It has been shown [2] that:

$$s^2 = \frac{1}{N-1}\left[\sum_{i=1}^{N} (X_i - \overline{X})^2\right]$$, is an unbiased estimator

for σ^2, s^2 may be conveniently calculated by:

$$s^2 = \frac{1}{N-1}\left[\sum_{i=1}^{N} X_i^2 - \overline{X}\sum_{i=1}^{N} X_i\right]$$

To obtain an estimate of the confidence we may place on s^2, we assume that in the large sample case, as here, the distribution of s^2 is normal in

which case:

$$\mathrm{Var}(s^2) = \frac{2\sigma^4}{N-1}, \quad \Pr\left\{\left[\, |s^2 - \sigma^2| \right] < k_\alpha \sqrt{\frac{2\sigma^4}{N-1}}\right\} \simeq \beta$$

ie

$$\frac{s^2}{1 + k_\beta \sqrt{\frac{2}{N-1}}} < \sigma^2 < \frac{s^2}{1 - k_\beta \sqrt{\frac{2}{N-1}}}$$

where k_β is the value of the standard normal r.v. which will have a probability of being exceeded of β .

The confidence interval here was chosen to be 99% or .99 which implies

$k_\beta = 2.57$

ie. $$\Pr\left\{ N(0,1) < 2.57 \right\} = 0.99$$

Thus the confidence range C was constructed:

$$C = s^2\left(1 - \frac{1}{k_\beta \sqrt{\frac{2}{N-1}}}\right)$$

C indicates the region about s^2 in which one has a probabilily greater than .99 of finding the true σ^2.

If we form C/s^2 we have

$$\frac{C}{s^2} \times 100\% = \frac{k_\beta}{1 + k_\beta \sqrt{\frac{2}{N-1}}} \times 100\%$$

In the data analysed here $k_\beta = 2.57$, $N = 3900$ and hence $C/s^2 =$ 5.42%.

This means that we are confident that the true σ^2 will lie within 5.4% of the estimated $\sigma^2 = s^2$ with probability .99.

Note that $$\lim_{N \to \infty} \frac{C}{s^2} = k_\beta$$

In each of the runs it was desired to plot both the signal and its' auto-correlation function. The units of the signal are converted to volts by dividing all data samples by 409.6 or $\frac{2048 \text{ levels}}{5 \text{ volts}}$.

Sine, square, and triangular waves were analysed to check the scaling accuracy. It was found that the accuracy of measurements exceeded that of the oscilloscope used to measure the input waves.

The D.C. signal was analysed to observe the amount of digitizing noise and hum introduced by the process.

The maximum change in the digitized level was about 4 numbers or $\frac{4}{2048} \times 100\% = 0.2\%$ of the full scale. Over the period of 0.8 seconds the mean square power was 4.74×10^{-6} watts and the rms. voltage was 2.17×10^{-3} volts. (A change of one number -2.44 m volts)

Thus for a 3.025 volt signal, the noise and hum is 62.8 db below the signal.

The presence of a strong secondary peak in $R(\tau)$ at $\tau = 1/60^{th}$ sec., indicates the presence of a strong 60 cps component in the noise. Assuming that at $\tau = \frac{1}{60}$ sec. the noise is uncorrelated, then $R(1/60) = 2 \times 10^{-6}$ or 66.6 db. with respect to a 3.025 volt signal. Thus the digitizing noise is 3.3 db stronger than the hum. If hum could be completely eliminated, the noise power would then be -63.3 db below a 3.025 signal or -53.7 db below 1 watt.

The "white" noise and the band-limited white noise samples were taken primarily to check the whiteness of the noise generator. Measurement

of the $\frac{1}{\epsilon}$ point of the $R(\tau)$ curve indicates the generator output would be flat within 3 db out to 200 cps.

A considerable discrepancy between the rms voltage as measured with a VTVM, and the calculated value of $R(0)$ and $\bar{s^2}$, was observed. Some of the error is due to the motion of the needle, and some due to the fact that the averaging time of the meter was about twice that of the correlater. However the most signifigant factor was a faulty measurement probe.

6. RECOMMENDATIONS

No attempt was made to "fix" the installation, and hence some problems arose that would not have occurred in a permanent installation.

Noise and hum reduction was only achieved by careful grounding and the use of BNC type coaxial signal leads. The use of coaxial cable is recommended for all digitizing work, for the above reason.

A clock pulse was used to synchronize left and right channel data. Unfortunately, the act of switching Ampex tape recorders on or off in the record mode causes transients to be generated on all tracks. This is a characteristic observed on both the Ampex CP100 unit of E.E. department and the Ampex FR-100 unit of the Aeronautical Engineering Department.

The transient pulses may be treated as clock pulses by the program, causing loss of channel time synchronization.

The problem was partially overcome by removing the output of the pulse generator until the transients were past. This is satisfactory unless the first clock pulse is close to the transients. A standard interval of 20

seconds is recommended between the start of data and the first clock pulse, as a temporary restriction pending hardware modifications.

A more permanent solution would be to imcorporate a time delay in all record channels of the Ampex CP100 to allow transients to die out, when starting and stopping the unit, before energizing the amplifiers. This should prevent spurious clock pulses from being formed.

Despite great care, occasional spurious pulses appear on the clock channel, and may cause false sampling to take place. The use of the short optimal length pseudo-random code as a clock pulse, together with a matched receiving filter would correct this problem.

The system as implemented has a 2.5 Kc upper frequency and 4000 samples continous sampling capability. Since at best, samples may be taken every 190 μsecs., of which only about 50 μsecs. are chargeable to the A/D converter, the most likely place to begin the search for higher sampling rates is in the computer itself, rather than in a more expensive A/D converter. By deleting certain functions from program DIGITIZE, the sampling may be made faster but in no case is it possible to sample at intervals smaller than about 100 μsecs.

Table I(a). Program Digitize.

Cell	Contents	Code	Explanations
0000	0101	PTA	
1	0603	ADN03	
2	7064	JPI64	Jump to INITIAL
3	7500	EXF00	
4	2410	2410	Set Enable
5	7500	EXF00	
6	1401	1401	Call A/D Channel 1
7	7600	INA	INPUT
0010	4176	STI76	Store sample in (0076)
11	2076	LDD76	
12	~~3464~~ 3465	SBN65	Enough Samples yet?
13	6134	NZF34	If not, go to cell 0047
14	7500	EXF00	
15	2400	2400	Clear Enable
16	2074	LDD74	Load Current Run No.
17	4160	STI60	Store in cell 0133
0020	2067	LDD67	Load Spare ID.
21	4161	STI61	Store in cell 0134
22	2075	LDD75	Load Current Block No.
23	4162	STI62	Store in cell 0135
24	2073	LDD73	Load No. of Blocks/Run
25	4163	STI63	Store in cell 0136
26	7500	EXF00	
27	2111	2111	Call 163 M.T.U.
0030	7303	OUT03	Output from
31	0000	C	computed L.W.A.
32	6102	NZF02	to
33	0133	0133	0133
34	2246	LDF46	Set A-0137
35	4076	STD76	Reset running storage address
36	2075	LDD75	Enough Blocks yet?
37	3473	SBD73	
0040	6155	NZF55	If not, go to cell 0115
41	0401	LDN01	
42	4075	STD75	Reset block no.
43	5455	AOD55	Update total block count
44	3454	SBD54	Check if capacity exceeded
45	6153	NZF53	If not, go to cell 0120
46	6061	ZJF61	If not, go to cell 0127
47	5476	AOD76	Begin intersample delay loop

Table I(b). Program Digitize.

Cell	Contents	Code	Explanations
0050	2066	LDD66	Load intersample delay word
51	0701	SBN01	If not zero, go back 1
52	6501	NZB01	If zero, go to cell 0005
53	7056	JPI56	0760-M.T.U. block capacity
54	0760	0760	Total No. Blocks written
55	0000	C	address
56	0005	0005	address
57	0003	0003	
0060	0133	0133	Address of Current Run No. ID
61	0134	0134	Address of Spare ID of ID
62	0135	0135	Address of Cur. Block No. ID
63	0136	0135	Address of Tot. Blocks ID
64	0100	0100	Address of INITIAL
65	C	C	Address of last word of data
66	M	M	Intersample delay word
67	M	M	Spare ID anything
0070	M	M	Initial Run No. Set 1
71	M	M	No. runs desired
72	M	M	No. samples/block 76408
73	M	M	No. blocks/run
74	C	C	Current run no.
75	C	C	Current Block Number
76	C	C	Running Storage Address
77	C	C	JUMP CONTROL
0100	4077	STD77	BEGIN INITIAL
101	2200	LDC00	
102	0137	0137	Set "A"-0137
103	4076	STD76	Initialize running storage address
104	3072	ADD72	Compute last address
105	4031	STD31	Store in 0031
106	0701	SBN01	
107	4065	STD65	Store in 0065
0110	2070	LDD70	
11	4074	STD74	Initialize Run No.
12	0401	LDN01	
13	4075	STD75	Initalize Block No.
14	7077	JPI77	END INITIAL
15	5475	AOD75	Update Current Block No.
16	5455	AOD55	Update Current Block Count
17	7057	JPI57	Go to 0003

Table I(c). Program Digitize

Cell	Contents	Code	Explanations
0120	2074	LDD74	Have enough runs
21	3471	SBD71	been done yet?
22	6103	NZF03	If no, go to 0125
23	2074	LDD74	If yes, display last run no.
24	7701	HLT01	Halt 01
25	5474	AOD74	Update current run no.
26	7057	JPI57	Go to 0003
27	7500	EXF00	Call E.O.F.
0130	1111	1111	Write an EOF Mark
31	2055	LDD55	Load block count
32	7702	HLT02	Halt 02
33	C	C	RUN #ID
34	C	C	SPARE #ID
35	C	C	BLOCK #ID
36	C	C	BLOCKS/RUN ID.
0137	D	D	Data Storage
	D	D	
7776	D	D	Data Storage

Notes:

1) C is ENTRY PROGRAM COMPUTED
M is MANUAL ENTRY
D is DATA STORAGE

2) CDC 1604 Identifier will appear as follows:

Word 1	RUN #	Spare	Block #	Blocks/RUN	Format 016
Word 2	Sample 1	Sample 2	Sample 3	Sample 4	

etc.

Table II. Modifications to Digitize

Mod	Cell #	From	To	Effect of Modification
1	0126	7057	7703	Individual blocks clocked at end of a run. To restart run at 0003
2	0014 0015 0117 0126	7500 2400 7057 7057	0300 0300 7056 7703	Initial block clocked. Subsequent blocks taken as fast as possible. HLT in 0126. When run complete reset FFI manually
3	0014 0015 0040 0126	7500 7500 6155 7057	0300 0300 6113 7703	Initial block clocked. Subsequent blocks taken immediately. All blocks have same ID. HLT when run complete in 0126. FFI manually reset.
4	0013	6134	6506	Eliminates variable sampling delay. Samples possible in this mode every 120 μsec.

Table III. Program TEST 160

Note: M means Manual Entry

Cell	Contents	Code	Explanations
0000	7500	EXF00	Begin Bias-set
1	2410	2410	Set Enable
2	7500	EXF00	
3	1401	~~1410~~ 1401	Call A/D Channel 1
4	7600	INA	Input to "A" in Reg.
5	4070	STD70	Store "A" in 0070
6	7500	EXF00	
7	2401	2401	Call D/A Channel 1
0010	7303	OUT03	Output Cell 0070 to
11	0072	0072	Cell 0071 INC.
12	6102	NZF02	
13	0070	0070	
14	0400	LDN00	Load zero's.
15	4071	STD71	Store in 0071
16	6414	ZJB14	Jump back to 0000
17	7700	HLT00	HLT. End Bias-set
0020	7500	EXF00	Begin Timer
21	2410	2410	Set Enable
22	7500	EXF00	Call A/D Ch. 1
23	1401	1401	
24	7600	INA	
25	4143	STI43	Store Sample in 0043
26	0300	NOP	No Operation-NOP
27	0300	NOP	These are Time
0030	0300	NOP	Dummies to
31	0300	NOP	Match Program DIGITIZE
32	0300	NOP	Timing
33	0300	NOP	
34	0300	NOP	
35	0300	NOP	
36	2042	LDD42	Load Timing Control Word
37	0701	SBN01	Subtract 1
0040	6501	NXB01	If not zero, go back 1
41	7044	JPI44	If is zero, jump to 0022
42	M	M	Timing Control Word
43	0043	0043	
44	0022	0022	
45	7701	HLT01	End Timer

```
      SUBROUTINE DATA(IDENT,MAX,KLIST,KFLAG)
      DIMENSION IBLOCK(1001),KDATA(4000,2)
      COMMON KDATA, IBLOCK
C            ARG 1.  IDENT IS A 16 OCTAL DIGIT IDENTIFYING NUMBER
C                  SUPPLIED BY THE 160 PROGRAM WHEN TAPE WAS MADE.
             ARG 2.  MAX IS THE NUMBER OF SAMPLES/BLOCK.
C            ARG 3.  KLIST IS EITHER 1 OR 2 AND DECIDES WHICH SIDE OF
C                  KDATA YOU WANT THE DATA TO BE UNPACKED INTO
C            ARG 4.  KFLAG IS AN ERROR FLAG RAISED BY DATA. IF =0,NO ERROR
C            LOGICAL UNIT 1 IS USED BY DATA TO FIND BLOCK.
C            EACH TIME DATA IS CALLED IT FINDS THE DESIRED BLOCK ON THE TAPE,
C               AND UNPACKS IT INTO EITHER KDATA(M,1) OR KDATA(M,2)
C            IN THE CALLING PROGRAM DIMENSION KDATA(4000,2),  AND
C                  DECLARE KDATA COMMON.
C     KDATA IS THE OUTPUT LIST AND IS REFERENCED BY KLIST
C     SUBROUTINE DATA CALLS ON SR UNPACK AND SR FINDIT
C     AFTER UNPACK THERE EXISTS ONE 1604 WORD/160 WORD
  824 FORMAT(48H1 ERROR OCCURRED IN FINDIT SUBROUTINE AT HEADER )
  825 FORMAT(50X,O16)
  828 FORMAT(33H0 I HAVE UNPACKED DATA HEADED BY ,O16      )
  830 FORMAT( 1X,I4, 3X,O16)
      CALL FINDIT(IDENT,MAX,IFLAG)
      IF(IFLAG) 822,823,822
  822 PRINT 824 $ PRINT 825,IDENT $KFLAG=1$ GO TO 829
  823 JMAX=MAX/4+1
      CALL UNPACK (IBLOCK,JMAX,KLIST,KDATA(1,KLIST))
      KFLAG = 0 $ PRINT 828 , IDENT
      DO 826 M=1,4
  826 PRINT 830,M,KDATA(M,KLIST)
      J = MAX - 3
      DO 827 M = J, MAX
  827 PRINT 830,M,KDATA(M,KLIST)
  829 CONTINUE
      END
```

```
      SUBROUTINE FINDIT(IDENT,MAX,IFLAG)
      DIMENSION KDATA(4000,2),IBLOCK(1001)
      COMMON KDATA,IBLOCK
  814 FORMAT(54H0 A PARITY ERROR WAS DETECTED BUT RUN WAS NOT STOPPED )
  815 FORMAT(34H0 PARITY ERROR OCCURRED AT HEADER ,016)
  818 FORMAT(34H0 I HAVE LOCATED DATA HEADED BY    ,016)
  820 FORMAT(20H0 UNABLE TO LOCATE  ,016)
      MAX1=MAX/4 + 1 $ ASSIGN 811 TO JUMP
  805 BUFFER IN (1,1)(IBLOCK(1),IBLOCK(MAX1))
  806 IF(UNIT,1) 806,807,808,810
  807 IF(IDENT-IBLOCK(1)) 805,813,805
  808 GO TO JUMP,(811,812)
  809 PRINT814$PRINT815,IDENT$PRINT818,IDENT$IFLAG=0$GO TO 821
  810 IF(IDENT-IBLOCK(1)) 805,809,805
  811 REWIND 1$ ASSIGN 812 TO JUMP $ GO TO 805
  812 REWIND 1$ GO TO 819
  813 PRINT 818,IDENT $ IFLAG = 0 $ GO TO 821
  819 PRINT 820,IDENT $ IFLAG=1
  821 CONTINUE
      END
```

```
        IDENT        UNPACK           ARGS IBLOCK,JMAX,KLIST,KDATA
        ENTRY        UNPACK           SUBR UNPACK CALLED BY SR DATA
UNPACK  SLJ          **               +
        SIU     1    EXIT
        LIU     1    UNPACK           + JMAX=MAX/4+1, MOST=JMAX-1
        LDA     1    0                GETS ADDR OF CALLING ARGS
        SAL          U1               +U1 IS JMAX ADDR
        ARS          24               IBLOCK PACKED IN DATA 1001 WDS
        INA          -1               +
U1      SAU          AADRS            ADRS OF IBLOCK IN AADRS
        LDA          **               +FILLED IN BY 3 INSRUCTIONS BACK
        SAU          BADRS            STORE JMAX IN LOOP COUNT
        LDA     1    1                +GO GET ADRS OF NEXT ARG
        SAU          J4               +STORE ADRS OF KDATA(1,KLIST)
        INA          1                ADVANCE ADRS
        SAL          J3               +STORE ADRS OF KDATA(2,KLIST)
        INA          1
        SAU          J2               +DITTO FOR KDATA(3,KLIST)
        INA          1
        SAL          J1               +DITTO FOR KDATA(4,KLIST)
        INI     1    2                SET CORRECT EXIT ADRS
        SIU     1    EXIT+1           +STORE IN EXIT INST
        SIL     2    EXIT             SAVE INDEX 2
        ENI     1    2                +FIRST DATA WORD IN IBLOCK
        ENI     2    0                FIRST WORD IN KDATA IS DATA
AADRS   LDA     1    **               +IBLOCK(J)
        LRS          12               SHIFT 4TH WORD INTO A REG
J1      QRS          36               +RIGHT JUST,SIGN EXTEND
        STQ     2    **               STORE IN KDATA (I+4,KLIST)
        LRS          12               +SHIFT 3RD WORD INTO Q
        QRS          36               RIGHT JUST ,SIGN EXTEND
J2      STQ     2    **               +STORE IN KDATA(I+3,KLIST)
        LRS          12               SHIFT 2ND WORD INTO A REG
J3      QRS          36               + RIGHT JUST, SIGN EXTEND
        STQ     2    **               STORE IN KDATA(I+2,KLIST)
```

```
J4        STA    2  **                   +1ST WORD NOW RT.JUST +SIGN EXT
          INI    2  4                    I=I+4
BADRS     ISK    1  **                   +ISK ON JMAX
          SLJ       AADRS                J=J+1 REPEAT LOOP
EXIT      ENI    1  **                   +RESTORE INDEX 1
          ENI    2  **                   RESTORE INDEX 2
          SLJ       **                   + JUMP OUT
          END                            SUBROUTINE UNPACK
```

```
-COOP,,BARRETT BOX B,I/1/0/49/S/1S/2S/E/45=54,10,10000.
-FTN,L,A,E.
      PROGRAM TEST
      DIMENSION KDATA(4000,2),XDATA(4000,2),IDENT1(12),IDENT2(12),ITITLE
     1(12),RTAU(100),KSHIFTS(100),SIGNAL(100),SIG2(100)
      COMMON KDATA
      EQUIVALENCE (KDATA,XDATA)
      TYPE REAL KSHIFTS
C     0001000000010002 AND 0001000000020002 8.0VP-P SQ WAVE 250 CPS
C     0002 0000 0001 0002 AND 0002 0000 0002 0002 5VP-P SAWTOOTH 250CPS
C     0003 0000 0001 0002 AND 0003 0000 0002 0002 6VP-P SINE  250 CPS
C     0004 0000 0001 0002 AND 0004 0000 0002 0002 -9VDC IF 4VOLTS NEG.
C     0005 0000 0001 0002 AND 0005 0000 0002 0002 -WHITE NOISE RC 20CPS
C     0006 0000 0001 0002 AND 0006 0000 0002 0002 -PURE WHITE NOISE
C     FACTOR OF -409.6 IS TO CONVERT A/D REPRESENTATION TO VOLT [illegible] GROUND
C     DATA IS ON RESERVE TAPE 182,4000 SAMPLES/BLOCK,5KC SAMPLING RATE
C     EXCEPT RUN 5, THE BAND-LIMITED WHITE NOISE.IS SAMPLED EVERY
C     794 MICROSECONDS WHICH IS  1/10 TH OF A TIME CONSTANT.
C     LOGICAL TAPE UNIT  1  MUST BE USED FOR INPUT.
 1000 FORMAT(3X,I3,3X,I4)
 1001 FORMAT(1H1 )
 1002 FORMAT(3X,O16,3X,O16)
 1003 FORMAT(6X,I4)
 1004 FORMAT(27H1 ERROR IN SR DATA AT IDENT,O16      )
 1005 FORMAT(68H0    IDENTIFIER              MEAN       SIGMA SQUARE CONFIDENC
     1E     SIGMA  )
 1006 FORMAT(3X,O16,2X,E12.4,1X,E12.4,1X,E12.4,1X,E12.4)
 1007 FORMAT(6A8)
      READ 1000,NUMBHDRS,MAX $PRINT 1001 $ PRINT 1000,NUMBHDRS,MAX
      DO 10  I=1, NUMBHDRS
   10 READ 1002,IDENT1(I),IDENT2(I) $ PRINT 1002,IDENT1(I),IDENT2(I)
      READ 1003,NOSHIFTS $ PRINT 1003,NOSHIFTS $ CALL TIME
C     DATA NOW ALL READ AND PRINTED BACK. 11 IS MAIN DO LOOP ON N
      XMAX=MAX $ MAXTAU= MAX-NOSHIFTS+1 $ XMAXTAU=MAXTAU
```

```
      DO 11 N=1,NUMEHDRS $ SUM1=0.  $ SUM2=0. $ SS1=0. $ SS2=0.
        PRINT 1001 $ IDENT=IDENT1(N) $KLIST=1
        CALL DATA (IDENT,MAX,KLIST,KFLAG)
        IF(KFLAG) 50,51,50
   50 PRINT 1004,IDENT $ GO TO 11
   51 IF(IDENT1(N)-IDENT2(N))52,53,52
   53 DO 12 J=1,MAX
      XDATA(J,1)=XDATA(J,1)$XDATA(J,1)=XDATA(J,1)/(-409.6)
      SUM1=SUM1+XDATA(J,1) $ SS1=SS1 + XDATA(J,1)**2
   12 CONTINUE
      XBAR1=SUM1/XMAX  $  DO 14 KK=1,MAX
      XDATA(KK,1) = XDATA(KK,1)-XBAR1 $ XDATA(KK,2)=XDATA(KK,1)
   14 CONTINUE
C     DATA NOW MEAN REMOVED AND CONVERTED TO VOLTS IF AUTOCORRELATION
C     WAS DESIRED
       SS2=SS1 $XBAR2 = XBAR1 $ SUM2 = SUM1 $ GO TO 55
   52 IDENT=IDENT2(N) $ KLIST=2  $ CALL DATA(IDENT,MAX,KLIST,KFLAG)
      IF(KFLAG) 50,54,50
   54 DO 13 J = 1,MAX
      XDATA(J,2)=XDATA(J,2) $ XDATA(J,1)=XDATA(J,1)
      XDATA(J,2)=XDATA(J,2)/(-409.6) $ XDATA(J,1)=XDATA(J,1)/(-409.6)
      SUM1=SUM1+XDATA(J,1) $ SUM2=SUM2+XDATA(J,2)
      SS1=SS1+XDATA(J,1)**2 $ SS2=SS2+XDATA(J,2)**2
   13 CONTINUE
      XBAR1 = SUM1/XMAX $ XBAR2 = SUM2/XMAX
      DO 15 J=1,MAX
      XDATA(J,1)=XDATA(J,1)-XBAR1 $ XDATA(J,2)= XDATA(J,2)-XBAR2
   15 CONTINUE
C     S**2 = 1/(N-1)(SUM OF XSUB1,SQUARED - XBAR*SUMOF XSUBI)
C     DATA NOW MEAN REMOVED AND CONVERTED TO VOLTS IF X-CORR WAS DESIRED.
   55 SS1=(SS1-XBAR1*SUM1)/(XMAX-1.0)$ SS2=(SS2-XBAR2*SUM2)/(XMAX-1.0)
      SIGMA1=SQRTF(SS1) $ SIGMA2= SQRTF(SS2)
      FACTOR=2.57*(SQRTF(2.0/(XMAX-1.0)))
      CONF1=SS1-(SS1/(1.0+FACTOR)) $ CONF2=SS2-(SS2/(1.0+FACTOR))
C     LINDGREN PAGE 191
```

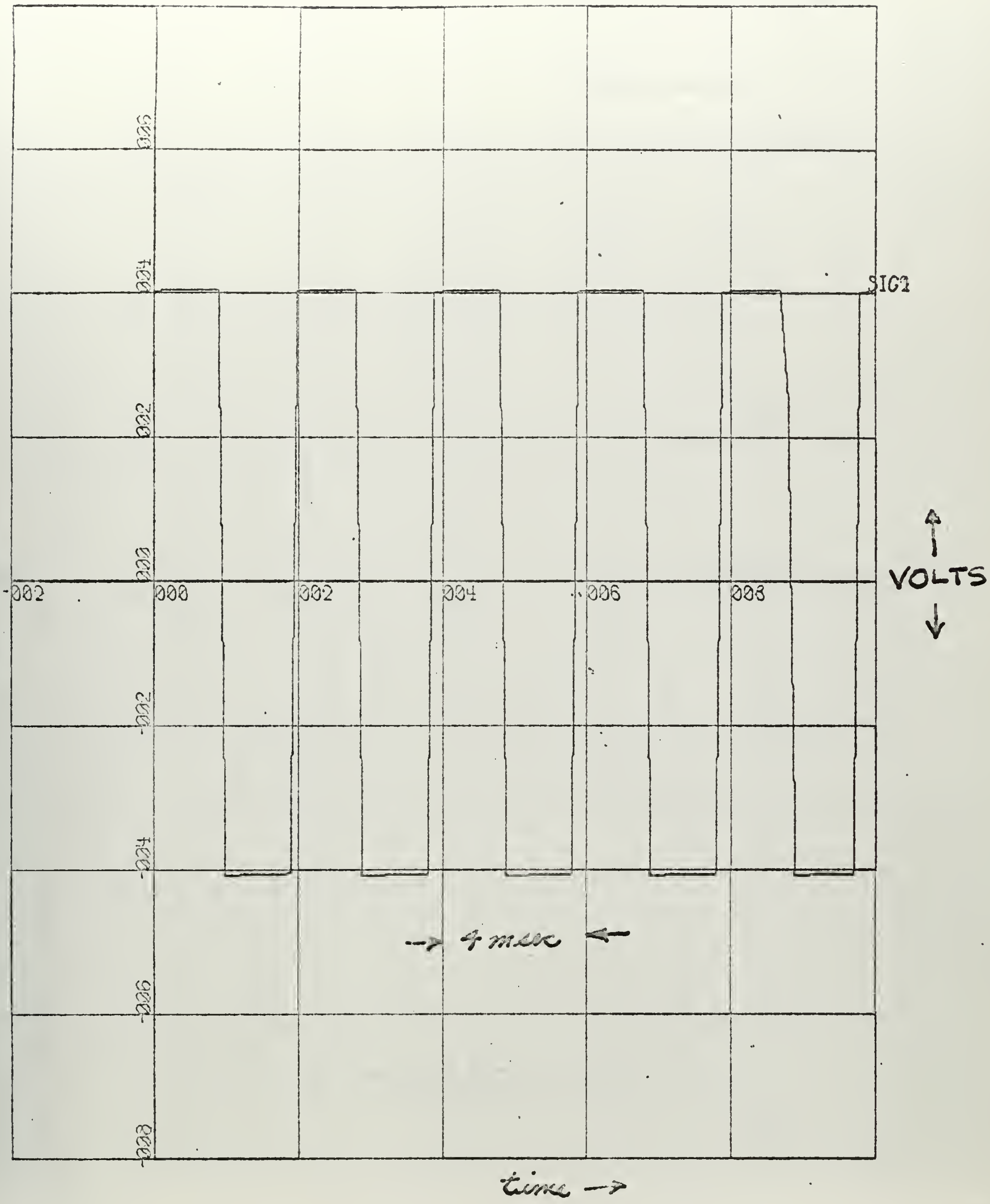
```
      PRINT 1005  $ PRINT 1006,IDENT1(N),XBAR1,SS1,CONF1,SIGMA1
      PRINT 1005  $ PRINT1006,IDENT2(N),XBAR2,SS2,CONF2,SIGMA2
C     DATA NOW READY FOR CORRELATION
      CALL TIME
      DO 16 L=1,NOSHIFTS
      SUM=0.0
      DO 17 K=1,MAXTAU
      J=K+L-1 $ SUM = SUM+XDATA(K,1)*XDATA(J,2)
   17 CONTINUE
      RTAU(L)=SUM/XMAXTAU  $ KSHIFTS(L)=L-1
      SIGNAL(L)=XDATA(L,1) $SIG2(L)=XDATA(L,2)
   16 CONTINUE
      CALL TIME
C     READY TO DRAW
      MOD=0  $ LABEL=4HRTAU  $ READ 1007,(ITITLE(I),I=1,6)
      READ 1007,(ITITLE(I),I=7,12)
      CALL DRAW(NOSHIFTS,KSHIFTS,RTAU,MOD,0,LABEL,ITITLE,0,0,4,1,2,2,6,8
     1,1,LAST)      $     MOD=1
      LABEL=4HSIG1$READ 1007,(ITITLE(I),I=1,6)$READ1007,(ITITLE(I),I=7,
     112) $ CALL DRAW(NOSHIFTS,KSHIFTS,SIGNAL,MOD,0,LABEL,ITITLE,0,0,4,1
     1,2,2,6,8,1,LAST) $ MOD=3 $ LABEL=4HSIG2
      CALL DRAW(NOSHIFTS,KSHIFTS,SIG2,MOD,0,LABEL,ITITLE,0,0,4,1,2,2,6,8
     1,0,LAST) $ CALL TIME
   11 CONTINUE
      END
```

```
6   4000
     1000000010002   0001000000010002
     2000000010002   0002000000010002
     3000000010002   0003000000010002
     4000000010002   0004000000010002
     5000000010002   0005000000010002
     6000000010002   0006000000010002
      10
     1000000010002 V 0001000000010002 P.T. 182
N BARRETT.SQUARE WAVE CORR.3900PTS 100LAGS
     1000000010002 RT.182- 250 CPS SQUARE WAVE
8 VOLTS P-P. 200 MICROSEC. SAMPLING   N BARRETT
     2000000010002 V 0002000000010002 RT. 182
N BARRETT.SAWTOOTH CORRELATION 3900PTS 100 LAGS
     2000000010002 RT.182 .250 CPS SAWTOOTH
5VOLTS P-P. 200 MICROSECS SAMPLING      N BARRETT
     3000000010002 V 0003000000010002 RT 182
N BARRETT.SINE WAVE CORRELATION 3900 PTS 100LAGS
     3000000010002 RT 182. 250 CPS SINE WAVE
6VOLT P-P. 200 MICROSEC SAMPLING        N BARRETT
     4000000010002 V 0004000000010002 RT 182
N BARRETT. -3 VOLTS DC CORRELATION.MEAN REMOVED
     4000000010002 RT 182  D.C VOLTAGE
N BARRETT MEAN HAS BEEN REMOVED
     5000000010002 V 0005000000010002 RT 182
N BARRETT 20CPS RC BANDLIMITED NOISE CORRELATION
     5000000010002 RT 182. 20CPS RC FILTERED
WHITE NOISE.794 MICROSEC SAMPLING       N BARRETT
     6000000010002 V 0006000000010002 RT 182
WHITE NOISE CORR. 7.07 V RMS SIGNAL     N BARRETT
     6000000010002 RT 182  WHITE NOISE  7.07V
RMS. 200 MICROSEC SAMPLING              N BARRETT
```

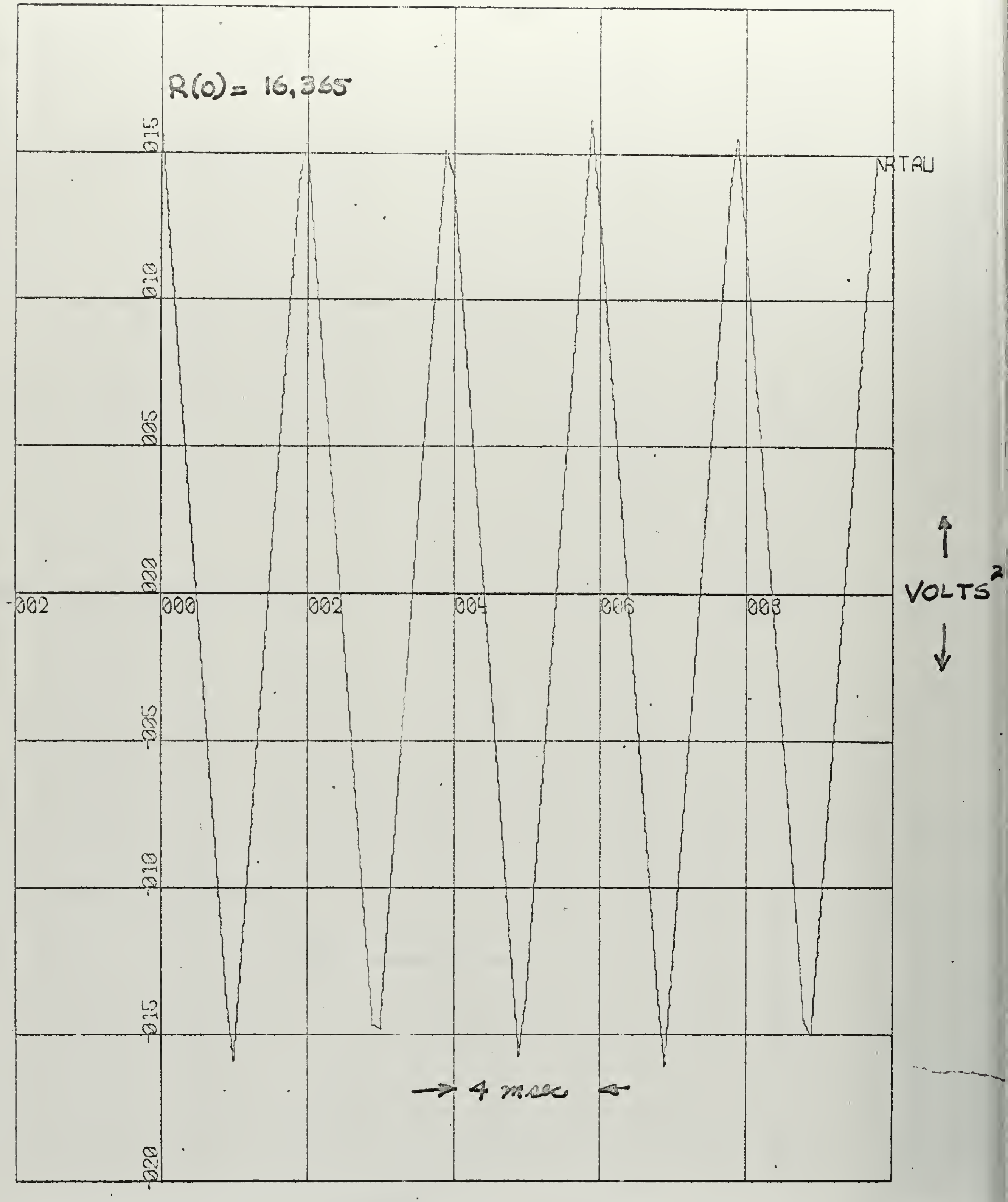
```
-COOP,,  BARRETT N  BOX B  ,I/1/0/3/S/1S/2S/E/2=57,15,10000.
-FTN,L,A,E.
      PROGRAM SHUFFLE
      DIMENSION MEMRY(1001),IDR(300),IDL(300)
C     THIS PROGRAM IS TO REORDER BLOCKS OF DATA ON TAPE SO THAT WHEN
C     TAPE IS READ BY SR DATA THE BLOCKS TO BE RECOVERED WILL BE IN
C     CONSECUTIVE ORDER. THIS WILL PREVENT TAPE HAVING TO REWIND BEFORE
C     GETTING NEXT BLOCK.
C        THE INPUT TAPE HAS BLOCKS ON IT IN ORDER IDR(1),IDR(2),...IDR(N)
C     FOLLOWED BY IDL(1),IDL(2),...IDL(N) .
C        THE OUTPUT TAPE HAS BLOCKS ON IT IN ORDER IDR(1),IDL(1),IDR(I),
C     IDL(2),        ,IDR(N),IDL(N)
C        IN THIS PROGRAM INPUT TAPE (UNORDERED) IS LOGICAL TAPE 1
C                           SCRAP TAPE (TEMP HOLD) IS LOGICAL TAPE 2
C                           OUTPUT TAPE (ORDERED ) IS LOGICAL TAPE 3
C     IDR IS A 16 OCTAL DIGIT NUMBER IDENTIFYING BLOCKS OF DATA MADE
C     ON THE RIGHT CHANNEL. IDL IS IDENTIFYING THE LEFT CHANNEL
C     N IS THE NUMBER OF PAIRS OF ID S .M IS NUMBER OF SAMPLES/BLOCK.
C     MEMRY IS A TEMPORARY HOLDING ARRAY.
C     IF THE PROGRAM CANNOT FIND ONE OF THE PIECES OF DATA YOU SAID WAS
C     ON TAPE INPUT , THE PROGRAM GENERATES A BLOCK OF ZEROS AND PUTS IT
C     ON THE OUTPUT TAPE . THIS IS WHAT IS IMPLIED BY THE STATEMENT,
C     CANNOT FIND DATA,RUN CONTINUES.
C     IF HOWEVER, THE PROGRAM CANNOT FIND A PIECE OF DATA THAT IT HAS
C     ALREADY WRITTEN ON TAPE 2 , THE PROGRAM EXITS.
C
    1 FORMAT(3X,I3,3X,I4)
    2 FORMAT(3X,O16.3X,O16)
    3 FORMAT(1H1)
    4 FORMAT(36H0 NUMBER OF PAIRS OF IDENTIFIERS IS ,I3   )
    5 FORMAT(42H0 DESIRED ORDER OF HEADERS ON OUTPUT TAPE   )
    8 FORMAT(25H0 INPUT CONTROL DATA END       )
    9 FORMAT(29H0 NUMBER OF SAMPLES/BLOCK IS ,I4 )
   10 FORMAT(36H0 PARITY ERROR OCCURRED ON TAPE 1 AT ,O16,3X,I3       )
   11 FORMAT(19H1 UNABLE TO LOCATE ,O16,3X,I3         )
   12 FORMAT(50H1 TAPE 2 NOT ABLE TO LOCATE IDR(K).RUN TERMINATED ,O16,3
     1X,I3   )
```

```
13 FORMAT(36H0 PARITY ERROR OCCURRED ON TAPE 2 AT ,O16,3X,I3  )
14 FORMAT(50H1 TAPE 1 NOT ABLE TO LOCATE IDL(K).RUN CONTINUES  ,O16,3
  1X,I3  )
C
   READ 1,N,M $ PRINT 3 $ PRINT 4,N $ PRINT 9,M $ PRINT 5
   DO 20 J=1,N
   READ 2,IDR(J),IDL(J) $ PRINT 2,IDR(J),IDL(J)
20 CONTINUE
   PRINT 8 $ M=M/4+1 $ PRINT 3
C  CONTROL DATA NOW IN
C  PUT RIGHT CHANNEL ON TAPE2
C
   DO 22 J=1,N  $ IPAR=0
   ASSIGN 38 TO JUMP
44 BUFFER IN (1,1) (MEMRY(1),MEMRY(M))
40 IF(UNIT,1)40,41,42,42
41 IF(IDR(J)-MEMRY(1)) 44,31,44
42 GO TO JUMP,(38,39)
38 REWIND1 $ ASSIGN 39 TO JUMP $ GO TO 44
43 IF(IDR(J)-MEMRY(1)) 44,30,44
30 PRINT 10 ,IDR(J),J
31 BUFFER OUT (2,1)(MEMRY(1),MEMRY(M)) $ GO TO 22
39 REWIND 1 $ PRINT 11,IDR(J),J $ GO TO 37
37 MEMRY(1)=IDR(J) $ DO 36 L=2,M
36 MEMRY(L)=0
   GO TO 31
22 CONTINUE
   END FILE 2 $ REWIND 2
C  TAPE 2 NOW HAS RIGHT CHANNEL BLOCKS IN ORDER
C
   DO 23 K=1,N
   ASSIGN 52 TO JUMP  $  ASSIGN 54 TO JUMP1
46 BUFFER IN(2,1)(MEMRY(1),MEMRY(M))
47 IF(UNIT,2)  47,48,49,50
48 IF(IDR(K)-MEMRY(1))46,51,46
```

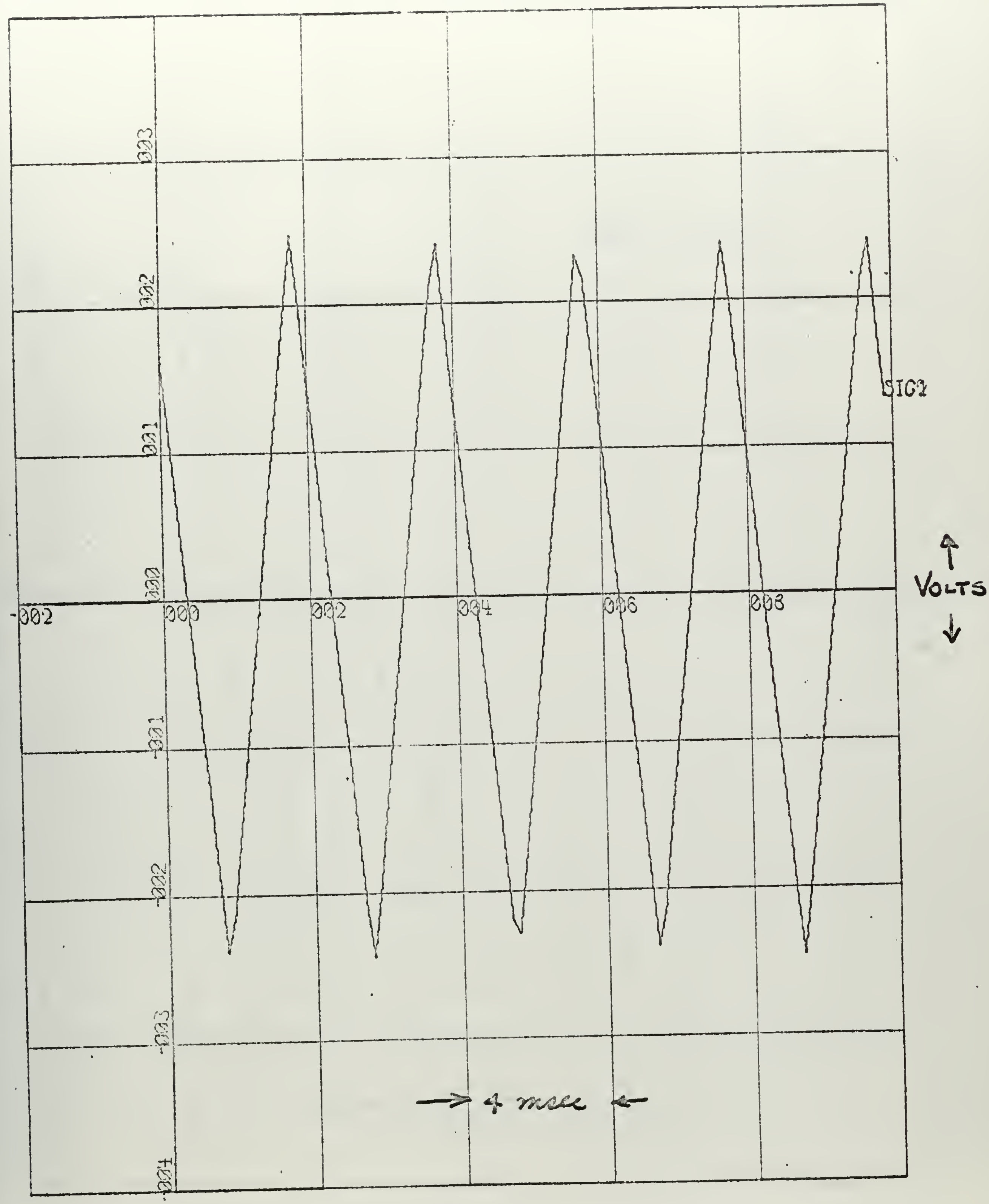
```
49 GO TO JUMP,(52,53)
52 REWIND 2 $ ASSIGN 53 TO JUMP $ GO TO 46
53 REWIND 2 $ PRINT 12,IDR(K),K $ K=N $ GO TO 23
50 IF(IDR(K)-MEMRY(1))46,64,46
64 PRINT 13,IDR(K),K $ GO TO 51
51 BUFFER OUT(3,1)(MEMRY(1),MEMRY(M))
60 BUFFER IN (1,1)(MEMRY(1),MEMRY(M))
56 IF(UNIT,1)56,57,58,59
58 GO TO JUMP1,(54,55)
54 REWIND 1 $ ASSIGN 55 TO JUMP1 $ GO TO 60
55 REWIND 1 $ PRINT 14,IDL(K),K $ GO TO 61
61 MEMRY(1) = IDL(K) $ DO 62 L = 2,M
62 MEMRY(L)=0
66 BUFFER OUT(3,1)(MEMRY(1),MEMRY(M)) $ GO TO 23
59 IF(IDL(K)-MEMRY(1))60,65,60
57 IF(IDL(K)-MEMRY(1))60,66,60
65 PRINT 10,IDL(K),K $ GO TO 66
23 CONTINUE
   END FILE 3 $ END FILE 3 $ END FILE 3
   REWIND 1 $ REWIND 2 $ REWIND 3
   END
      END
      FINIS
-EXECUTE.
  60   4000
   1002100010024   0004003200010024
       ETC.,ETC., DOWN TO
   1002100240024   0004003200240024

   2002100010024   0005003200010024
       ETC.,ETC., DOWN TO
   2002100240024   0005003200240024

   3002100010024   0006003200010024
       ETC.,ETC., DOWN TO
   3002100240024   0006003200240024
```



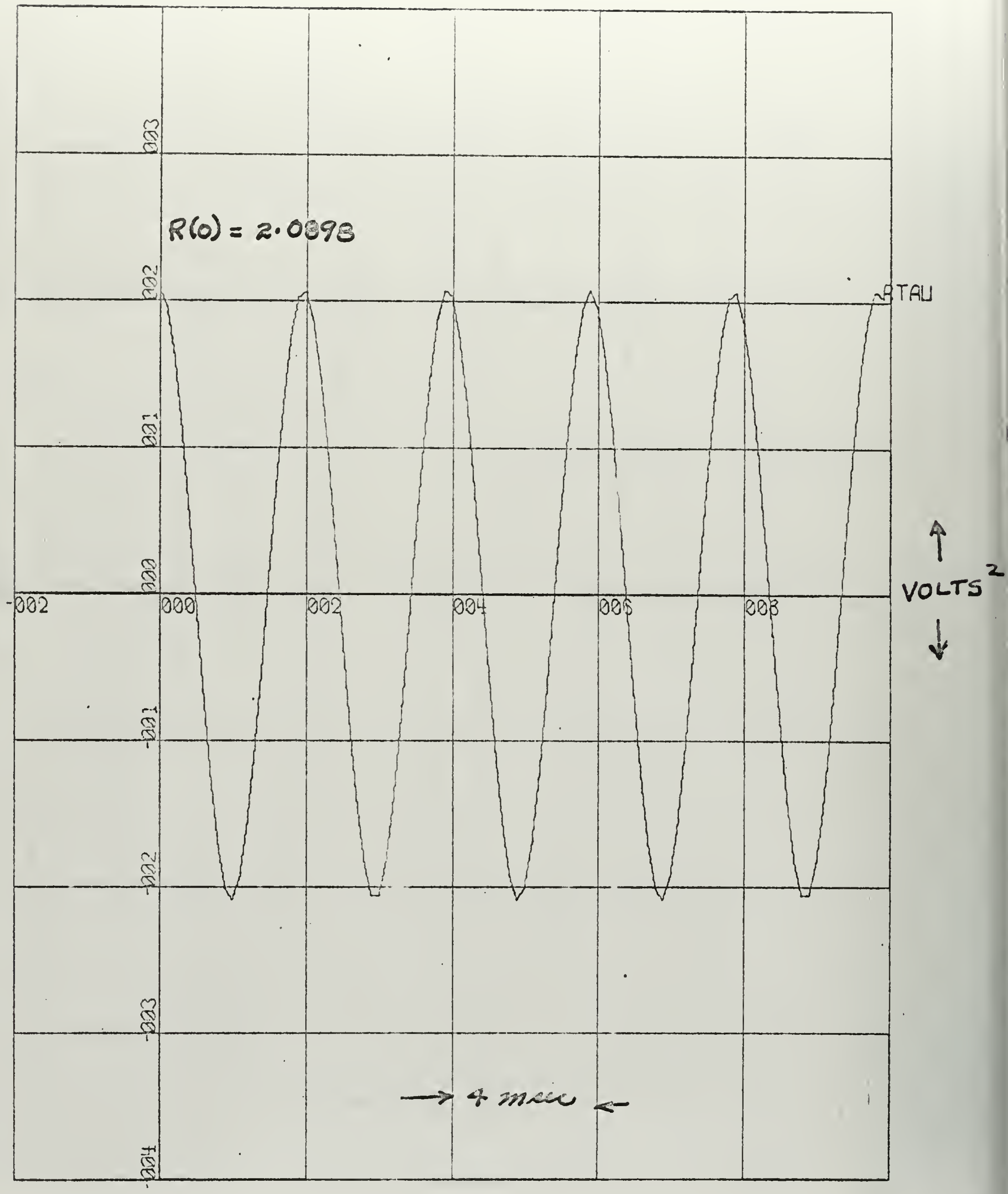
X Scale-20 units/inch. Y Scale- 2 unit/inch.
Figure 7- 253 cps square wave, 8 volts p-p. 200 usec. sampling.



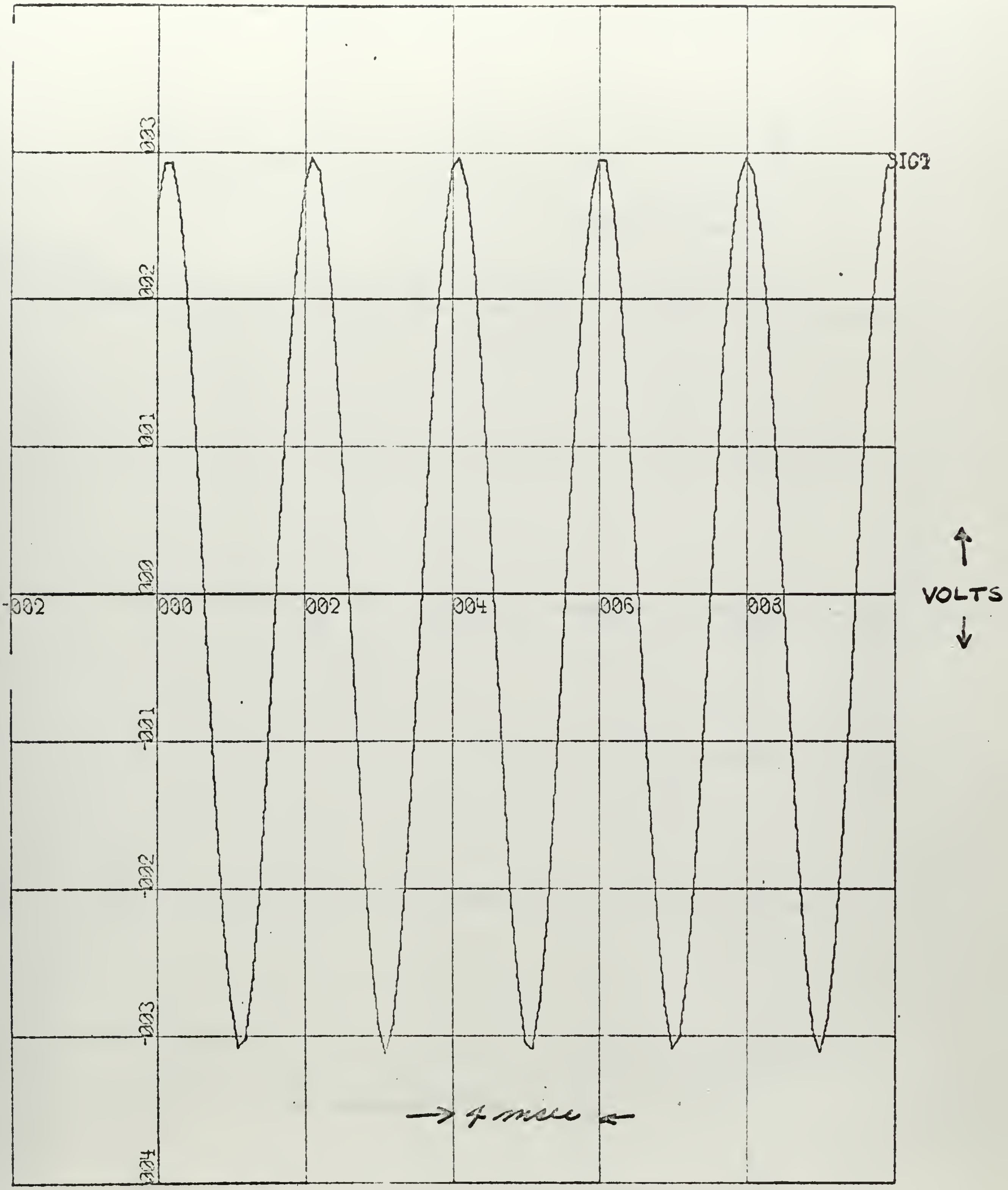
X Scale-20 units/inch. Y Scale- 5 units/inch.
Figure 8- Auto-correlation of Figure 7 over 3900 samples.



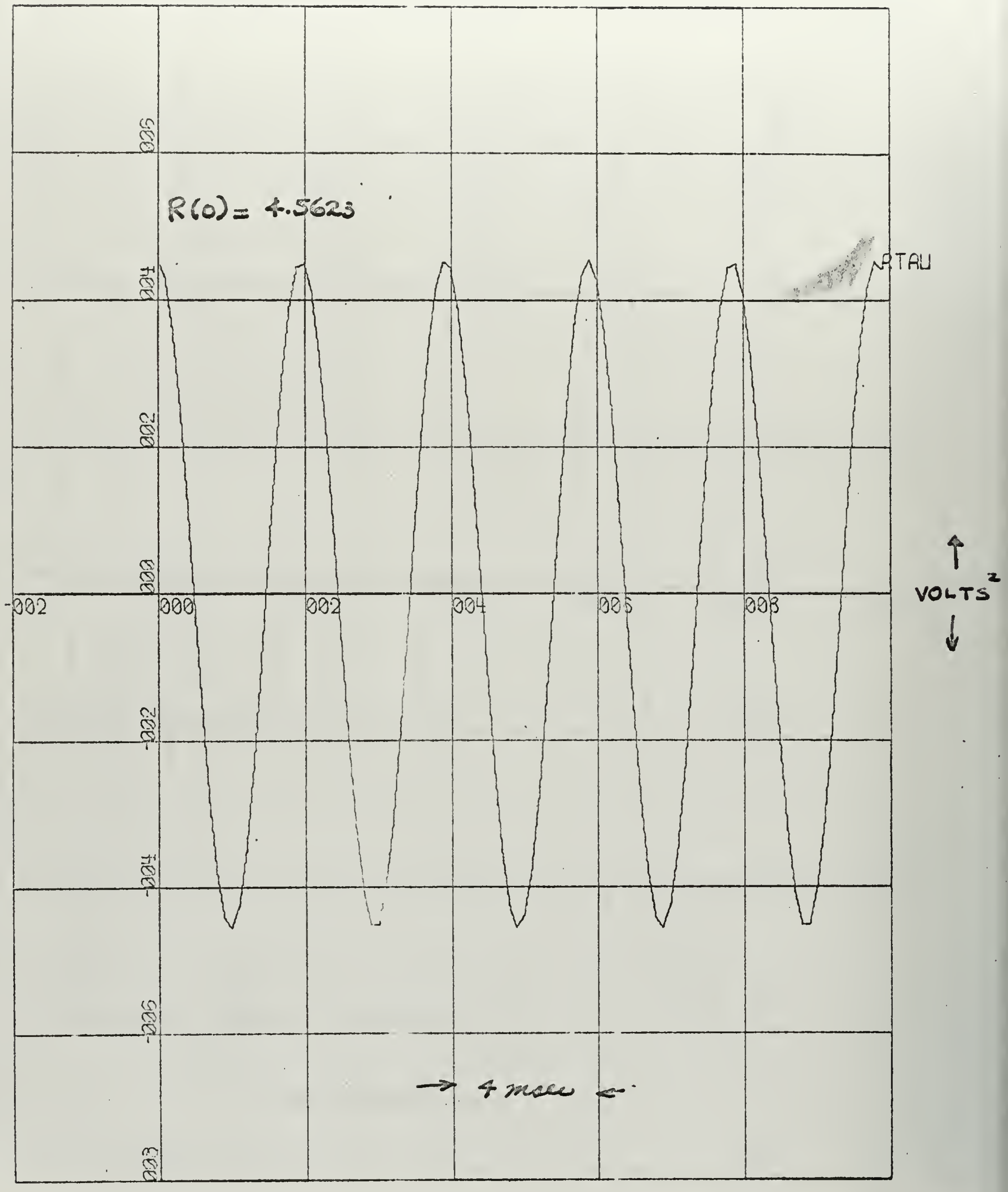
X Scale- 20 units/inch.   Y Scale-1 unit/inch.
Figure 9- 253 cps sawtooth wave, 4.8 volts p-p.
200 usec. sampling.



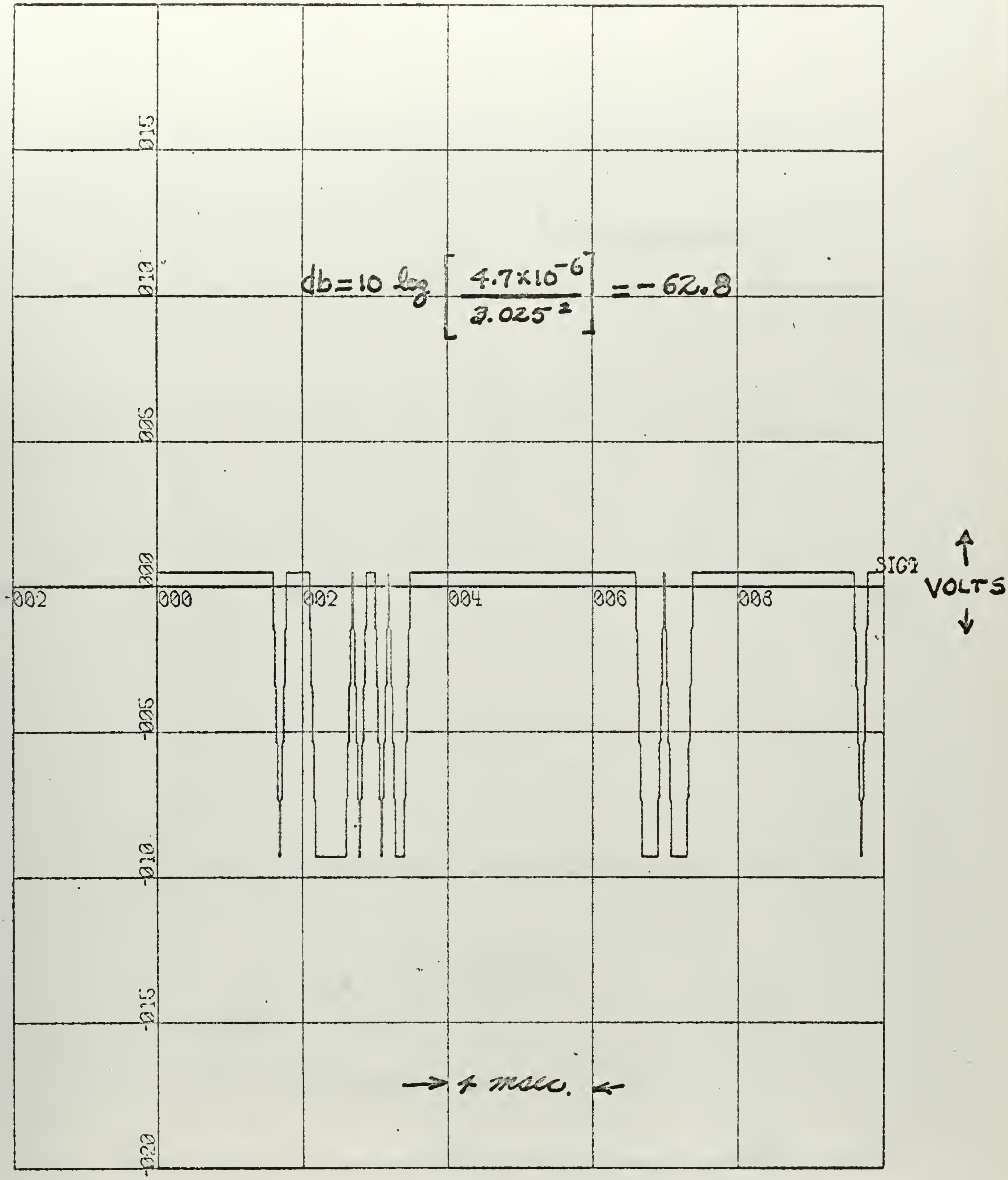
X Scale- 20 units/inch. Y Scale-1 unit/indh.
Figure 10- Auto-correlation of Figure 9 over 3900 samples.



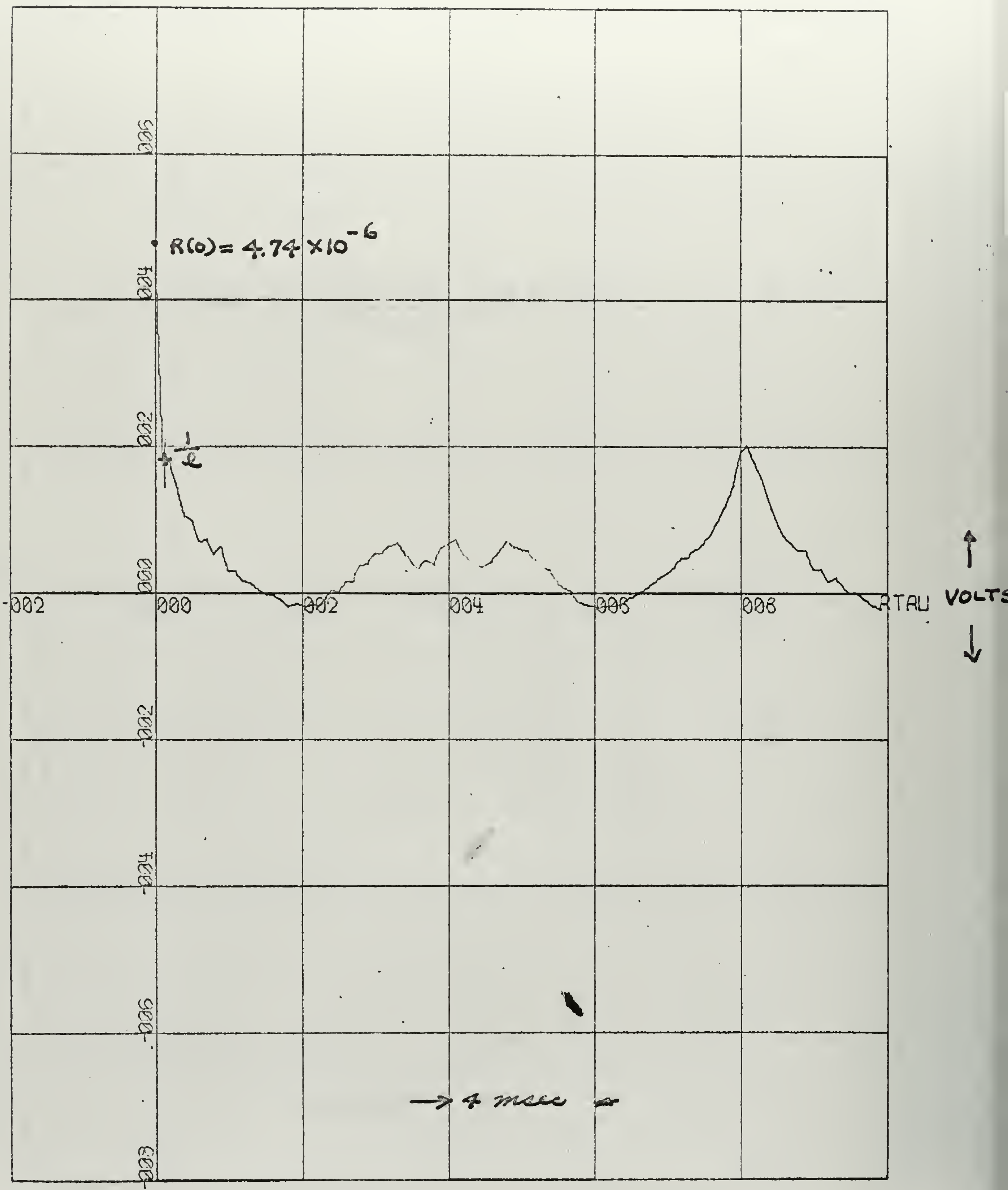
X Scale- 20 units/inch. Y Scale- 1 unit/inch.
Figure 11- 253 cps sine wave at 6 volts p-p.
200 usec. sampling.



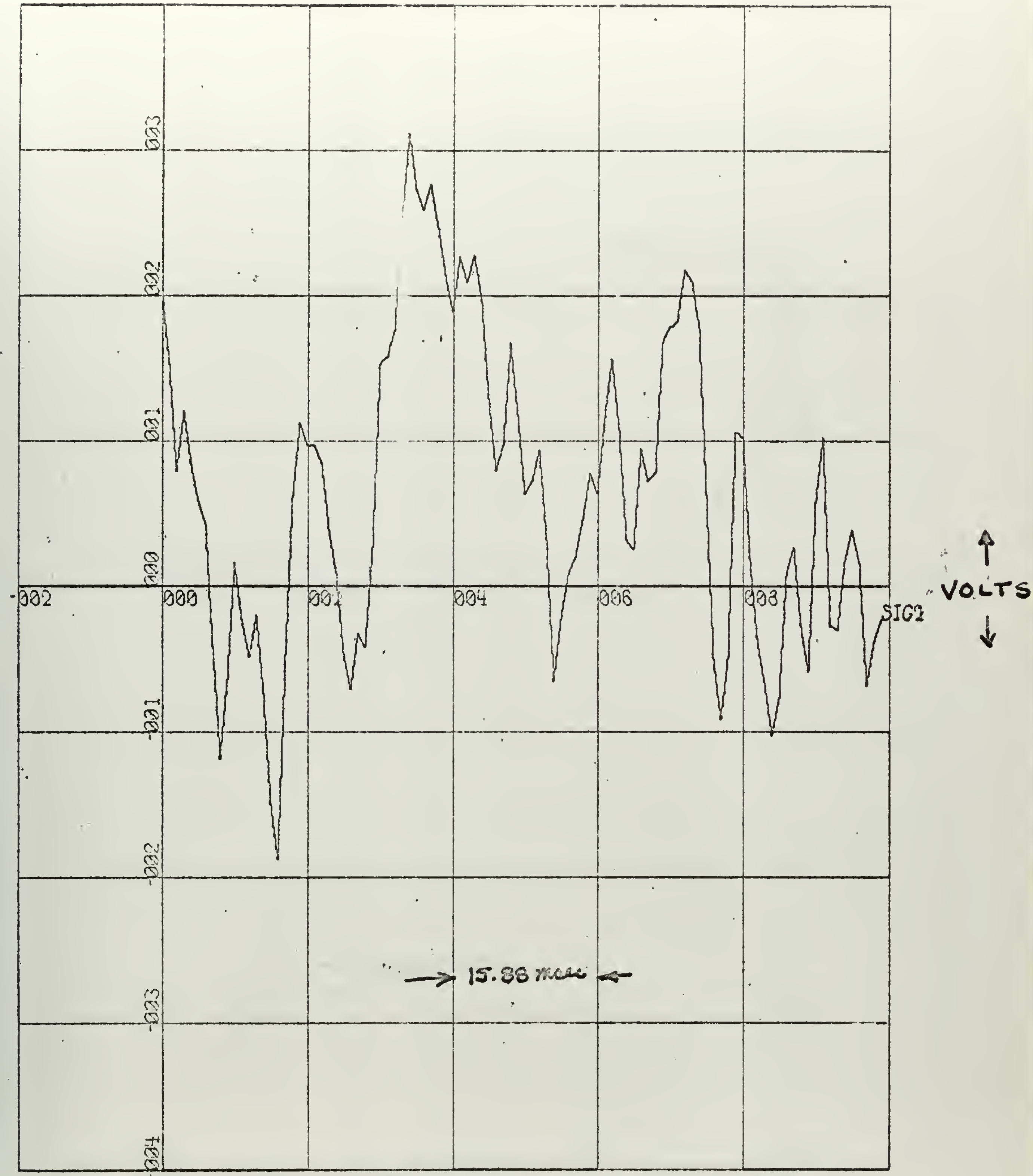
X Scale- 20 units/inch. Y Scale- 2 unit/inch.
Figure 12- Auto-correlation of Figure 11 .
3900 samples .



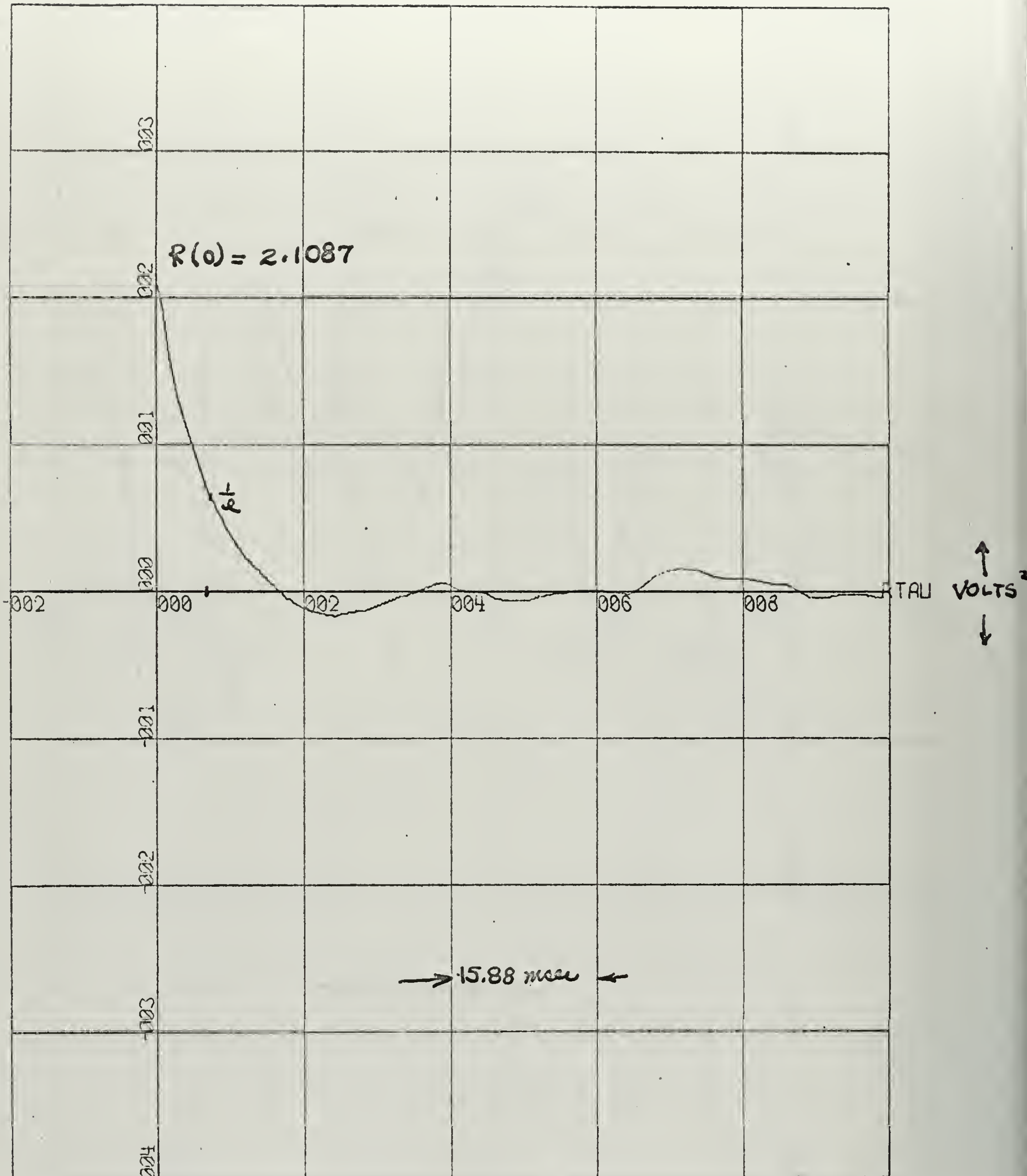
X Scale- 20 units/inch. Y Scale- 0.005 units/inch.
Figure 13- Fluctuations about the mean of a D.C. wave.



X Scale- 20 units/inch. Y Scale- 2×10^{-6} units/inch.
Figure 14- Auto-correlation of Figure 13 .
3900 samples .

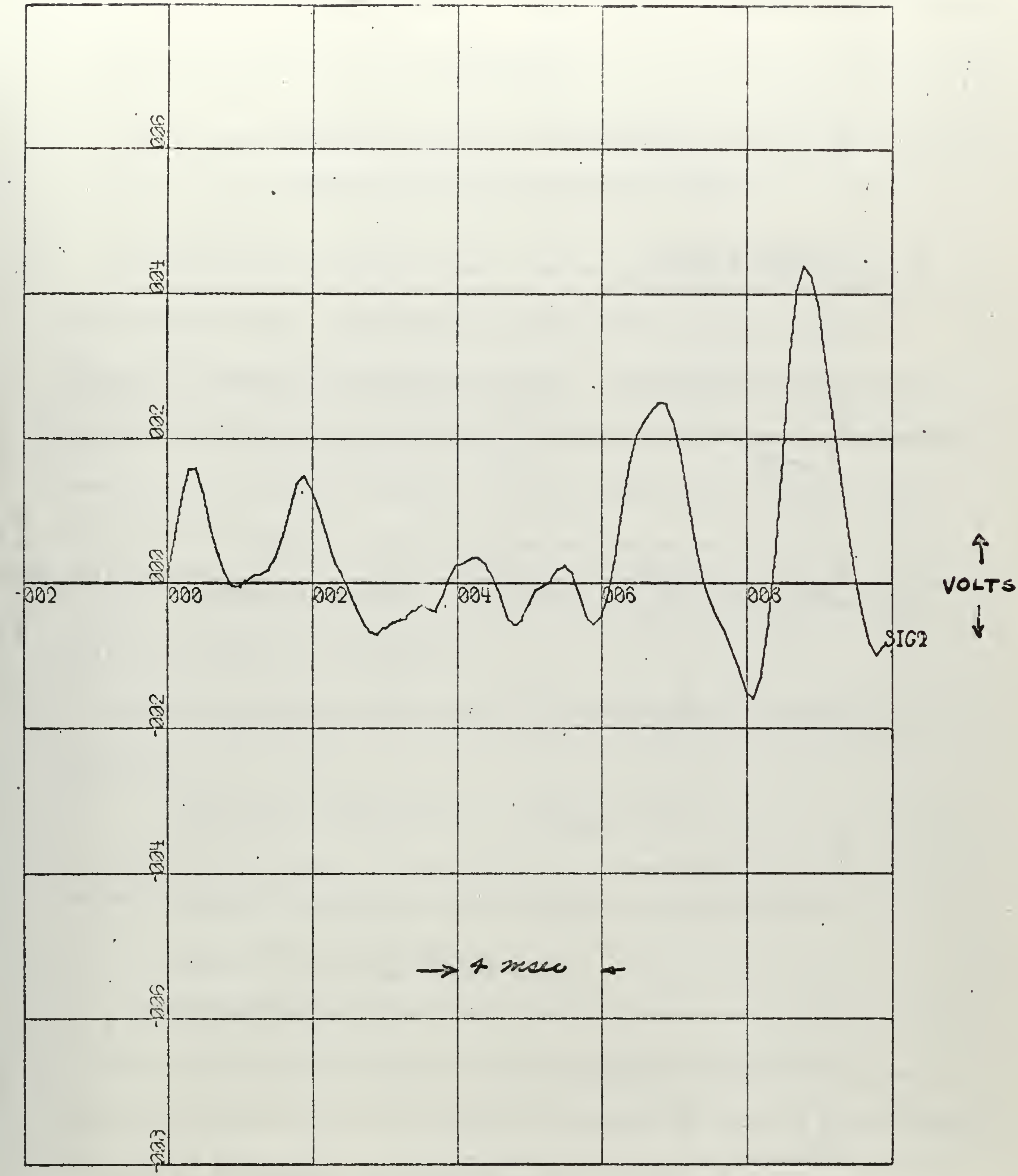


X Scale- 20 units/inch. Y Scale- 1 unit/inch.
Figure 15- Noise generator band-limited at 30 cps.

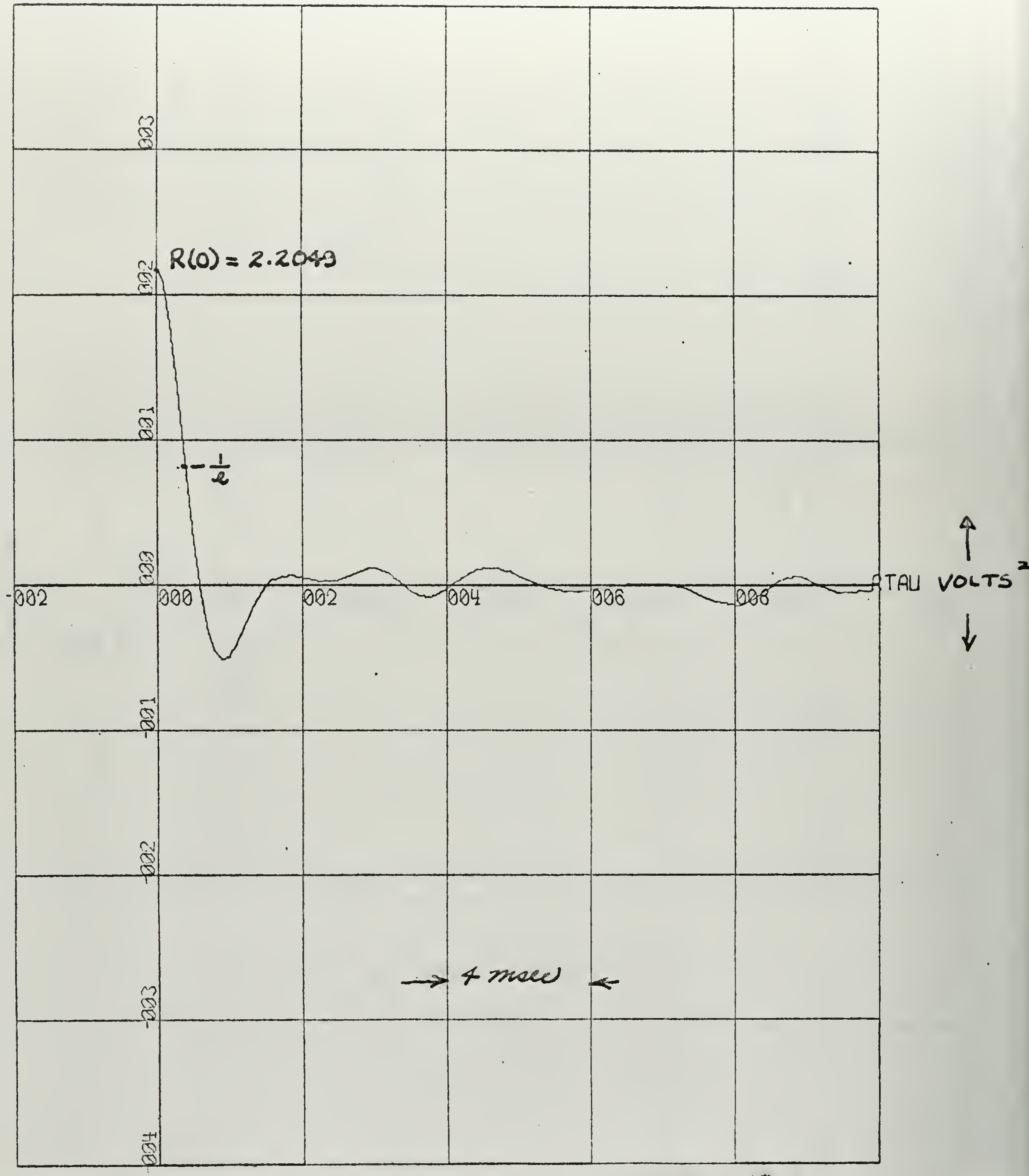


3 db. cutoff frequency of R/C filter is 29.5 cps.

X Scale- 20 units/inch. Y Scale- 1 unit/inch.
Figure 16- Auto-correlation of Figure 15.
3900 samples .



X Scale- 20 units/inch. Y Scale- 2 units/inch.
Figure 17- Noise generator output. 200 usec, sampling.



3 db. CUT-OFF FREQUENCY OF FILTER = 199 CPS.

X Scale- 20 units/inch. Y Scale- 1 unit/inch.
Figure 18- Auto-correlation of Figure 17:
3900 samples .

APPENDIX B

NOISE REMOVAL IN CORRELATION, USING A PRIORI KNOWLEDGE OF NOISE

Assume that as apriori information we are given the expected form of the interfering noise, represented by a normalized correlation function $\hat{R}^*_{NN}(k\Delta T)$ formed in the absence of signal. We assume that when the received signal is corrupted by noise, the statistics of the received noise are described by this priori knowledge. Specification of the statistics of the noise by the correlation function implies that the noise distribution is adequately described by only it's first two moments. We shall assume that the observed date is of mean zero.

Knowing the noise, it was shown that under certain assumptions we could write

$$\hat{R}_{SS}(k\Delta T) = \hat{R}^*_{S+N}(k\Delta T) - \alpha \hat{R}^*_{NN}(k\Delta T) \qquad (1)$$

where $\hat{R}^*_{S+N}(k\Delta T)$ is the observed normalized correlator output

$\hat{R}_{SS}(k\Delta T)$ is to be determined.

$\hat{R}^*_{NN}(k\Delta T)$ is known apriori

Obviously we must determine α before $R_{SS}(k\Delta T)$ may be determined. Since the correlator produces a correlation function the author's first thoughts were to find α using the observed and known correlation functions.

In the general case, the form of the correlation function of the signal is not know. For the class of input signals having a non-periodic correlation

function, any choice of α which forces the output to be periodic, ie $\max\limits_{\tau \neq 0} R = R(0)$, would be fallacious.

Therefore we shall only consider the class of signals having periodic correlation functions.

Let us consider the function

$$F(\alpha) = \left\{ \left(\hat{R}^{*}_{S+N}(0) - \alpha \hat{R}^{*}_{NN}(0) \right) - \max_{\tau \neq 0} \left| R_{S+N}(\tau) - \alpha R^{*}_{NN}(\tau) \right| \right\} \quad (2)$$

which may be written

$$F(\alpha) = \max_{\tau \neq 0} \left\{ (1 - \alpha) - \left| \hat{R}^{*}_{S+N}(\tau) - \alpha \hat{R}^{*}_{NN}(\tau) \right| \right\} \quad (3)$$

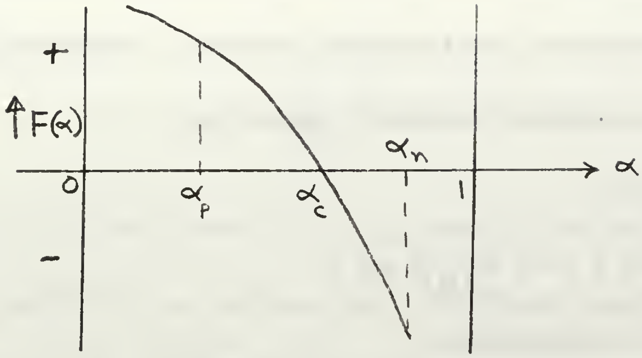
Since the functional forms of $\hat{R}^{*}_{S+N}(\tau)$ and $\hat{R}^{*}_{NN}(\tau)$ are not known explicitly no further analytic evaluation can be made of $F(\alpha)$.

Certain features of $F(\alpha)$ are known however in the case of periodic correlation function. If $F(\alpha) < 0$, then α is too large, and the output is not a valid correlation function since $R(\tau \neq 0) > R(\tau = 0)$. If $F(\alpha) > 0$, not enough of the noise correlation has been subtracted, and the output will be excessively noisy for small τ.

The best choice of α would then seem to be α_c such that $F(\alpha_c) = 0$.

The determination of α_c must be done iteratively since no explicit analytic formulation for the function is known.

Consider Figure 1 below



Initially set $\alpha_p = 0$, $\alpha_n = 1$, and $\alpha_i = \frac{\alpha_p + \alpha_n}{2}$

Evaluate $F(\alpha)$ and decide

a) $\alpha_p = \alpha_i$ if $F(\alpha_i) > 0$

b) $\alpha_n = \alpha_i$ if $F(\alpha_i) < 0$

c) Terminate if $|F(\alpha_i)| < \epsilon$

Then $\quad \alpha_{i+1} = \frac{\alpha_p + \alpha_n}{2}$

Having found α_c such that $-\epsilon \leq F(\alpha_c) \leq +\epsilon$ the portion of $\hat{R}^*_{VV}(\tau)$ due to signal may be estimated by substitution in Eq. (1), that is

$$\hat{R}_{SS}(\tau) = \hat{R}^*_{N'N'}(\tau) - \alpha_c \hat{R}^*_{NN}(\tau) \tag{4}$$

At this point we have a combinational ratio which causes a periodic output. However there is no guarantee that the new correlation function formed in Eq. (4) is the result of a physically realizable linear system. This is because the condition that the correlation be no larger for any τ other than $\tau = 0$ is a necessary condition and not a sufficient condition. A sufficient condition for realizability is that its transform, the power spectral density, have no negative portions.

Rather than attempting to determine α in the correlation domain, it is more fruitful to do a similar search in the frequency domain. The obvious advantage is that physical realizability is easily satisfied.

Transforming Eq. (1) we have

$$P_{SS}(k\Delta f) = P^*_{S+N}(k\Delta f) - \alpha\, P^*_{NN}(k\Delta f) \tag{5}$$

where the asterick indicates normalization.

An iterative search, similar to the previous search, results in determination of α_c subject to physical realizability constraints.

This is ensured by terminating the search when for any k, the value of Eq. (5) is less than .00001.

If any value greater than this is used then the contribution will be negative and realizability is violated. If a smaller value is used, more noise will be left than in the case $\alpha = \alpha_c$.

This value of $\alpha = \alpha_c$ was then used in Eq. (5) and in Eq. (1).

The results of Eq. (5) were renormalized to have unity area by the trapezoidal rule, that is

$$\frac{1}{2}(kp_1 + kp_L) + 2 \sum_{2}^{L-1} kp_i = 1 \tag{6}$$

from which we have

$$k = 2/\left(p_1 + p_L + 4 \sum_{2}^{L-1} p_i\right) \tag{7}$$

The results of Eq. (1) were renormalized by a factor $1/\hat{R}_{SS}(0)$.

To evaluate the results assume that the correlation of the desired signal formed in the absence of noise $\breve{R}^*_{SS}(k\Delta T)$ is available for comparison.

An estimate of the noise power in the correlator output may be found by evaluating

$$N_o = \frac{1}{K-1} \sum_{k=1}^{K} \left(\hat{R}_{SS}(k \Delta T) - (1 - \alpha_c) \tilde{R}^*_{SS}(k \Delta T) \right)^2 \tag{8}$$

The signal power in the correlator output may be evaluated several ways. One method is to assume that the scaled version of the pure signal, $(1 - \alpha) \cdot \tilde{R}^*_{SS}(\tau = 0)$, represents the peak value of the periodic output. In the case of a single sinusoid signal, the output power would be given by $((1 - \alpha) \cdot \tilde{R}_{SS}(0))^2/2$.

The input signal-to-noise power ratio may also be determined in several ways. One method is to compare the variance of the noise with that of signal plus noise to give

$$(S/N)_{INPUT} = (\hat{\sigma}^2_{S+N} - \hat{\sigma}^2_N)/\hat{\sigma}^2_N \tag{9}$$

An alternate method to determine S/N ratios and processing gains achieved is via the normalized power spectral density, obtained from the transform of the correlation function. This method is felt to be more accurate than determinations made from the correlation function because of the difficulties encountered in separating the signal components of a correlation function from the noise components.

Determination of signal to noise ratio from the normalized power spectral density proceeds as follows.

The energy under the normalized density curve, with respect to a unit change of index i, is given by

$$E = \int_{1}^{M+1} X \; di = 1 \tag{10}$$

Using the trapezoidal rule we may evaluate this as

$$\hat{E} = \frac{1}{2}(X_1 + X_{M+1}) + 2 \sum_{\substack{i=2 \\ i \neq S}}^{M} X_i + 2X_S = 1 \pm \epsilon \tag{11}$$

where X_S is the component due to the signal.

The desired output, Y, is given in terms of frequency rather than unit change of index and is given by

$$Y_i = k\, X_i / \hat{E} \tag{12}$$

where $k = 2 \Delta t M$

$\Delta f = k^{-1}$

Δt = interval between samples in seconds

M = the number of frequency estimates

Substitution of Eq. (12) in Eq. (11) yields

$$\hat{E} = N + S \tag{13}$$

where $N = \frac{1}{2}(X_1 + X_{M+1}) + 2 \sum_{\substack{i=2 \\ i \neq S}}^{M} X_i$

$S = 2 \hat{E} Y_S \Delta f$

The ratio of energy due to signal S, to that of noise, N is

$$\frac{S}{N} = \frac{2 \hat{E} Y_S}{\hat{E} - 2\hat{E} Y_S} = \frac{2 Y_S}{1 - 2 Y_S \Delta f} \tag{14}$$

or in decibels

$$(S/N)db = 10 \log_{10} (S/N) \tag{15}$$

2. Comparison of Auto and Cross-Correlation

No method of processing can improve the processing gain given by cross-correlation, which is taken to mean comparison of a signal plus noise with the completely specified input signal. This fact arises because of the a-priori information assumed to be available.

When in general the desired signal is not known, auto-correlation, that is comparison of a waveshape with a time shifted replica of itself, may allow detection of the signal. In no case does it do it as efficiently as cross-correlation with an uncorrupted signal.

If the noise can be assumed to be completely specified by it's correlation function, then it seems reasonable to assume that results approaching that of cross-correlation might be obtained by suitable removal of the known noise effects.

Lee [1] has developed models to estimate the processing gains possible for both auto and cross correlation, for the case of a single sinusoid $E_m \sin(wt+\theta)$ corrupted by noise of zero mean and variance σ_n^2.

For a single sinusoid input the desired output of the correlator $R_{SS}(\tau) = E^2 \cos w_o \tau$ where $E = Em/\sqrt{2}$.

At the input to the auto-correlator, let N_i be the input noise in rms value, and let

$$\rho_i = \frac{N_i}{S_i} = \frac{\sigma_n}{E} \qquad (16)$$

be the input noise-to-signal ratio. This definition while not conventional leads to compact formulation.

The output noise in rms value may be shown [1] to be

$$No_a = \left\{ \frac{1}{N} \left(\frac{E^4}{2} + 2E^2 \sigma_n^2 + \sigma_n^2 \right) \right\}^{1/2} \tag{17}$$

where N is the number of data samples considered. For sinusoidal input, the rms signal output

$$So_a = E^2 / \sqrt{2} \tag{18}$$

At the output of the correlator the signal to noise ratio R_{oa} is given by

$$R_{oa} = 20 \log_{10} \frac{S_{oa}}{N_{oa}} \tag{19}$$

Substitution of Eq. (13), (14) and (15) into Eq. (16) yields

$$R_{oa} = 10 \log_{10} \frac{N}{1 + 4\rho_i^2 + 2\rho_i^4} \tag{20}$$

If in the case of cross-correlation we assume the signal plus noise is correlated with a local signal Em sin w t, the output noise in rms value may be shown to be [1].

$$N_{oc} = \left\{ \frac{1}{N} \left(\frac{E^4}{2} + E^2 \sigma_n^2 \right) \right\}^{1/2} \tag{21}$$

and the output signal rms

$$S_{oc} = \frac{E^2}{\sqrt{2}} \tag{22}$$

The gain in db, R_{oc}, is given by

$$R_{oc} = 20 \log_{10} \frac{S_{oc}}{N_{oc}} \text{ db} \tag{23}$$

This may be written using Eq. (16)

$$R_{oc} = 10 \log \frac{N}{1 + 2\rho_i^2} \tag{24}$$

Inspection of Eq. (20) and Eq. (24) clearly shows the superiority of cross-correlation especially for large ρ_i (ie weak signals).

The superiority is expressed by the difference

$$G = R_{oc} - R_{oa} = 10 \log_{10} \frac{1 + 4\rho_i^2 + 2\rho_i^2}{1 + 2\rho_i^2} \tag{25}$$

The concept of processing gain is perhaps more useful for comparison of any proposed method with these two standard methods.

Let us define processing gain as the difference in signal to noise ratio at the output and the input of the processor.

$$\text{ie} \quad G_{pa} = R_{oa} - 10 \log_{10} \frac{E2}{\sigma_n^2} \text{ db}$$

$$= 10 \log_{10} \frac{N \rho_i^2}{1 + 4\rho_i^2 + 2\rho_i^4} \simeq 10 \log_{10} \frac{N}{4 + 2\rho_i^2} \tag{26}$$

$$\text{and} \quad G_{pc} = R_{oc} - 10 \log_{10} \frac{E^2}{\sigma_n^2} \text{ db}$$

$$= 10 \log \frac{N \rho_i^2}{1 + 2\rho_i^2} \simeq 10 \log_{10} \frac{N}{2} \tag{27}$$

Finally the difference in processing gains D_{ac}

$$D_{ac} = G_{pa} - G_{pa} = 10 \log_{10} (2 + \rho_i^2) \text{ db} \tag{28}$$

The above forms are quite accurate for $\rho_i > 3$. However the quantity $\rho_i = \frac{N_i}{S}$ rms is not the most commonly used description of signal to noise ratio. Accordingly equations (26) and (27) have been recast in terms of input signal to noise power ratios in db, and plotted in Figure 2. Eq. (28) was similarly treated and is shown in Figure 3.

Figure 3 conveniently shows the extent to which auto-correlation processing is able to emulate cross-correlation processing.

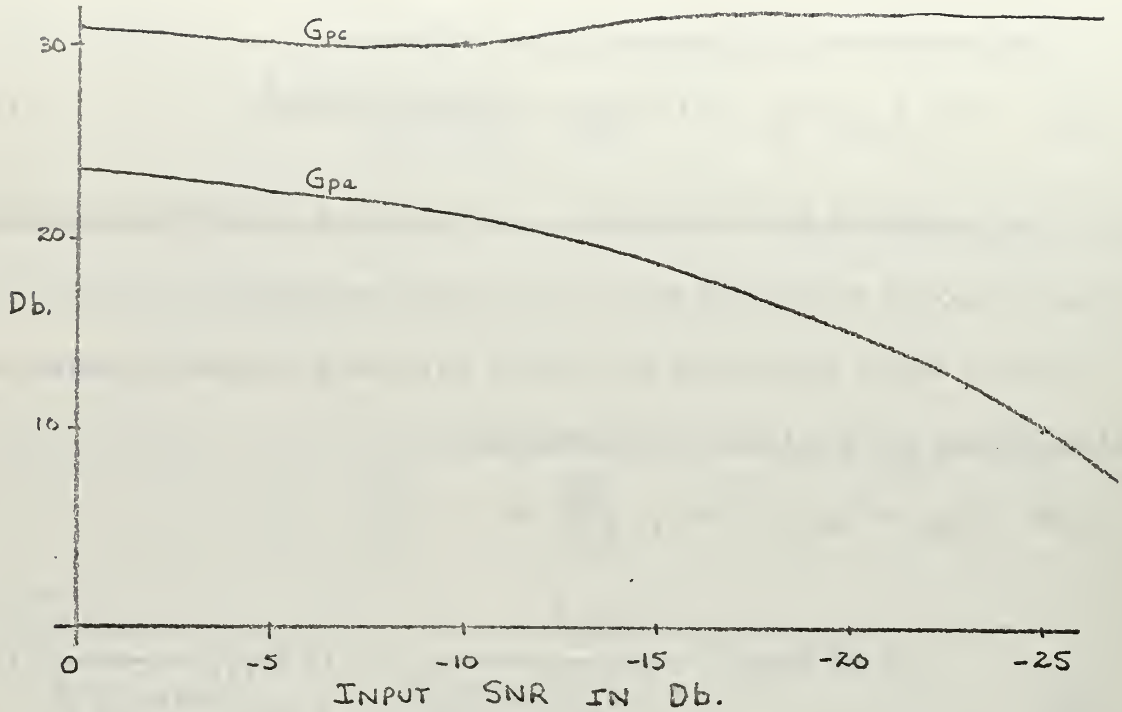


FIGURE 2: PROCESSING GAIN VS. SNR

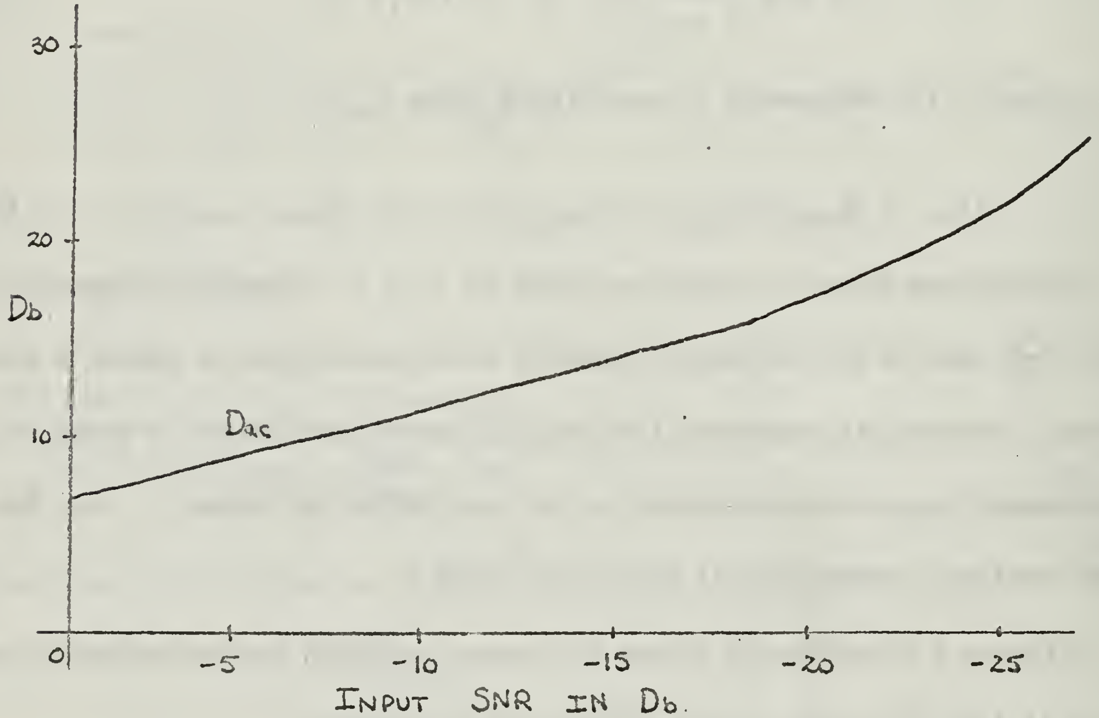


FIGURE 3: PROCESSING ADVANTAGE VS. SNR

Figures 2 and 3 are for 3750 samples.

3. Experimental Results

The results obtained for the case of unity signal to noise ratio are excellent, as may be seen by comparing Figures 5 and 10. Figure 5 is the power spectral density of a sine wave formed without noise. Figure 11 is the power spectral density of the 0 db signal to noise ratio input after the noise has been removed. A processing gain of 26 db was attained, as computed from the correlation function. This is mid-way between the performance of an auto-correlator and a cross-correlator.

Since results were so encouraging, the -10 db signal to noise case was then considered. Several problems became evident that previously were masked by the strong signal component.

Determination of α becomes difficult at low signal-to-noise ratios due to the small signal component in the auto-correlation function, and led to the formation of physically unrealizeable correlation functions, as evidenced in Figures 13, 14 and 15. Inspection of Figure 12, the power spectral density of the -10 db signal plus noise before noise removal, shows a strong component near 50 cps. It's transform, the correlation function (not shown) is difficult to interpret.

This suggests that the signal-to-noise ratio measure of quality may not be the most meaningful for this type of signal detection.

A more meaningful measure is the ability to distinguish a signal component in the power spectral density from a large noise spike. This ability

may be expressed in db as

$$A = 10 \log_{10} \frac{y_s}{\max y_i} \quad \text{for } i \neq s \tag{29}$$

If A is computed before and after noise removal, the increase in A is a direct measure of increased gain.

In other words, at low signal-to-noise ratios the problem shifts from one of interpreting a correlation function, to that of evaluation of the psd.

During the course of experiment it became evident that the spectrum of the noise may vary quite considerably from block to block. Since it was desired to remove the expected noise spectral density, more than a single block of noise must be used. By averaging the correlation function of noise over ten blocks much smoother spectra resulted (Figures 6, 7 and 8). The average noise correlation over ten blocks was used for the -10 db case. This of course implies stationarity over the period of time represented by 10 blocks of noise.

The claim that averaging separately formed correlation functions is equivelent to an increase in integration time, when the process is ergodic, should be verified. Examination of Eq. (26) and Eq. (27) leads to the conclusion that G_{pa} should increase by 3db when the number of samples is doubled, ie

$$\Delta G_{pa} = 10 \log_{10} \frac{N_1}{N} \tag{30}$$

where N_1 and N are the number of samples used.

This goal was not reached, but was closely approached, falling short by about 1 to 1.5 db.

The results obtained for the -10 db signal to noise ratio input are shown

in Figures 17 and 19, before noise removal, and in Figures 20 and 21 after noise removal. Noise removal had the effect equivalent to increasing the input signal to noise ratio by 14.46 db. Discrimination against noise peaks was increased by 3.42 db.

```
-COOP,,  BARRETT N  BOX B  ,I/1/0/49/S/1S/2S/E/45=54,20,10000.
-FTN,L,E,P.
      PROGRAM SIMSIG
      COMMON KDATA,IBLOCK
      DIMENSION ERROR(150)
      DIMENSION IBLOCK(1001)
      DIMENSION RSS(150),RNN(150),BEST(150),V2(150)
      DIMENSION KDATA(4000,2),IDENT1(20),IDENT2(20),RTAU(500),KSHIFTS(50
     10),RBAR(500),XLAG(500),RBARPT(500),ITITLE(12),XDATA(4000,2)
      EQUIVALENCE (KDATA,XDATA)
      TYPE REAL KSHIFTS
C
C  ORDER OF DATA CARDS--(NUMBHDRS,MAX),(IDENT CARDS),(NDSHIFTS),(NN),
C     (NINC),(IPSD),(IPRINT),(IPICT),(NORM),(IDRAW),(DELTAT)
C     (MISS) (SPIN) (WNIN) (FSNRIN) (FSFREQ) (NAMFSIG) (NAMENSE) (XK)
C     NUMBHDRS = NO. OF PAIRS OF HEADINGS
C     MAX =NO OF SAMPLES PER BLOCK
C     NN IS CONTROL ON WHAT NUMBER THRO YOU WISH TO PLOT RTAU
C     NINC IS INCREMENT ON N
C     IPSD IS FLAG.IF YOU WANT PSD SET TO 1
C     IPRINT IS FLAG.IF YOU WANT PSD PRINT OUT SET TO 1
C     RETREIVES TWO BLOCKS OF DATA AT A TIME. IF IDENT1(I)=IDENT2(I)
C     THEN PROGRAM AUTOCORRELATES ELSE CROSSCORRELATE
C
 1000 FORMAT (3X,I3,3X,I4)
 1002 FORMAT (3X,O16.3X,O16)
 1003 FORMAT(3X,I4)
 1009 FORMAT(31H1  ERROR IN SR DATA AT HEADING )
 1010 FORMAT(40X,I3,7X,O16)
 1012 FORMAT(1H1)
 1017 FORMAT(6A8)
 1060 FORMAT(55H0      IDENTIFIER         MEAN       SIGMA SQUARE    SIGMA)
 1061 FORMAT(3X,O16,2X,E12.4,1X,E12.4,1X,E12.4)
 1071 FORMAT(3X,I3)
 1072 FORMAT(45H FIRST OUTPUT  WHEN N = NN .         NN   =   I3/)
 1073 FORMAT(45H OUTPUT OCCURS WHEN N=NN+NINC    . NINC   =   I3/)
```

```
1074 FORMAT(45H IF IPSD=1,PSD DONE WHEN RTAU PLOTTED. IPSD= I3/)
1075 FORMAT(45H IF IPRINT=1,PSD WILL BE PRINTED. IPRINT =    I3/)
1076 FORMAT(45H IF IPICT =1,PSD WILL BE PLOTTED. IPICT  =    I3/)
1077 FORMAT(45H IF NORM  =1,DATA  NORMALIZED   . NORM   =    I3/)
1078 FORMAT(45H IF IDRAW=1 + NN-N=0,CORR.DRAWN . IDRAW  =    I3/)
1079 FORMAT(3X,F9.6)
1080 FORMAT(50H NO INSTRUCTIONS TO GIVE OUTPUT.PSD WILL BE DONE      )
1081 FORMAT(45H NUMBER OF IDENTIFIER PAIRS,  NUMBHDRS   =    I3/)
1082 FORMAT(45H NUMBER OF CORRELATION SHIFTS  ,NOSHIFTS =    I3/)
1083 FORMAT(45H NUMBER OF DATA SAMPLES/BLOCK ,   MAX    =    I4/)
1084 FORMAT(41H PSD ESTIMATES AT DELTAT SECONDS.DELTAT= F9.6 /)
1085 FORMAT(45H CORRELATION DONE SKIPPING SAMPLES BY MISS = I3/)
2000 FORMAT(14H MEAN OF BEST=F9.6,3X,19H MEAN SQUARE ERROR=F9.6/)
2001 FORMAT(4H AN=F12.6/)
2002 FORMAT(7H ARBAR=F12.6/)
2003 FORMAT(17H SCALE FACTOR AL=F9.6,2X,7H KOUNT=I2,2X,5H TOP=F9.6/)
2020 FORMAT(2X,F10.6)
2021 FORMAT(31H ESTIMATED SINEWAVE POWER IN = F9.6/)
2023 FORMAT(31H ESTIMATED NOISE POWER IN     = F9.6/)
2025 FORMAT(32H ESTIMATED INPUT POWER SNR DB = F10.6/)
2027 FORMAT(32H ESTIMATED SIGNAL FREQ IN CPS = I4/)
2028 FORMAT(3X,O16)
2029 FORMAT(31H IDENTIFIER FOR PURE SIGNAL  = O16/)
2030 FORMAT(31H IDENTIFIER FOR WHITE NOISE  = O16/)
2032 FORMAT(31H FACTOR BY WHICH SIG BELOW REF=F9.6/)
2033 FORMAT(21H CALCULATED SNR IN = F10.6)
2034 FORMAT(9H V2(1) = F9.6/)
2035 FORMAT(33H CALCULATED OUTPUT SIGNAL POWER= F9.6/)
2036 FORMAT(33H CALCULATED NOISE OUTPUT POWER = F9.6/)
2037 FORMAT(33H CALCULATED S/N RATIO OUT      = F10.6/)
2038 FORMAT(33H ESTIMATED PROCESSING GAIN   =   E12.4/)
2039 FORMAT(33H ESTIMATED PROCESSING GAIN DB  = F10.6/)
2040 FORMAT(22H ITERATION NUMBER N = I3/)
 701 FORMAT(10X,6F10.7)
```

```
      CALL TIME
      PRINT 1012
      M1=1 $ M2=6
      DO 700 J=1,21
       READ 701,(RSS(M),M=M1,M2)
      M1=M2+1 $ M2= M2+6
  700 CONTINUE
C     PURE SIGNAL CORRELATION NOW FORMED
      M1=1 $ M2=6
      DO 706 J=1,21
      READ 701,(RNN(M),M=M1,M2)
      M1=M2+1 $ M2=M2+6
  706 CONTINUE
C     AVG OF 10 NOISE CORRELATIONS NOW READ IN
      READ 1000,NUMBHDRS,MAX $ PRINT 1081,NUMBHDRS $ PRINT 1083,MAX
      DO 1001 N=1,NUMBHDRS
 1001 READ 1002,IDENT1(N),IDENT2(N)
      DO 1004 N=1,NUMBHDRS
 1004 PRINT 1002,IDENT1(N),IDENT2(N)
      READ 1003,NOSHIFTS $ PRINT 1082,NOSHIFTS
      READ 1071,NN $ PRINT 1072,NN
      READ 1071,NINC $ PRINT 1073, NINC
      READ 1071,IPSD $ PRINT 1074 ,IPSD
      READ 1071,IPRINT $ PRINT 1075,IPRINT
      READ 1071,IPICT $ PRINT 1076,IPICT
      READ 1071,NORM  $ PRINT 1077,NORM
      READ 1071,IDRAW $ PRINT 1078,IDRAW
      READ 1079, DELTAT $ PRINT 1084, DELTAT
      READ 1071,MISS $ PRINT 1085,MISS
      READ 1079,SPIN $ PRINT 2021,SPIN
      READ 1079,WNIN $ PRINT 2023,WNIN
      READ 2020,ESNRIN $ PRINT 2025,ESNRIN
      READ 1003,ESFREQ $ PRINT 2027,ESFREQ
      READ 2028,NAMESIG $ PRINT 2029,NAMESIG
      READ 2028,NAMENSE $ PRINT 2030,NAMENSE
```

```
      READ 1079,XK $ PRINT 2032,XK
C     DATA NOW ALL PRINTED BACK
      CALL TIME
      PRINT 1012
C     INITIALIZE RBAR
      DO 1005  KK = 1,500
 1005 RBAR(KK)=0.0
      S2AV1=0.0 $ S2AV2=0.0
      MAXTAU=MAX-NOSHIFTS*(MISS+1)+1+MISS $ XMAXTAU=MAXTAU
C
      DO 1006 N = 1,NUMBHDRS
C     MAIN DO LOOP ON WHOLE PROGRAM
      PRINT 1012
      IDENT=IDENT1(N)
      KLIST=1
      CALL DATA(IDENT,MAX,KLIST,KFLAG)
      IF(KFLAG)1007,1008,1007
 1007 PRINT 1009
      PRINT 1010,N,IDENT
      GO TO 9999
 1008 KLIST=2
      IF(IDENT1(N)-IDENT2(N))  1011,1014,1011
 1011 IDENT = IDENT2(N)
      CALL DATA(IDENT,MAX,KLIST,KFLAG)
      IF(KFLAG)1007,1014,1007
 1014 CONTINUE
      CALL TIME
C     TWO BLOCKS OF DATA ARE NOW RECALLED INTO KDATA1 OR KDATA2
C
      SUM=0.0  $ SS1=0.0
      DO 1020 I=1, MAX
      XDATA(I,1)=KDATA(I,1)
      XDATA(I,1)=XDATA(I,1)/(-409.6)
      SS1=SS1 + XDATA(I,1)**2
 1020 SUM=SUM + XDATA(I,1)
```

```
      XMAX = MAX
      XMEAN = SUM/XMAX
      SS1 = SS1/(XMAX-1.0) $ SIGMA1 = SQRTF(SS1)
      IF(NORM-1)1101,1100,1101
 1101 SIGMA1=1.0
 1100 PRINT 1060 $ PRINT 1061,IDENT1(N),XMEAN,SS1,SIGMA1
      DO 1021 I=1,MAX
 1021 XDATA(I,1)=(XDATA(I,1)-XMEAN)/ SIGMA1
      Y=N $ S2AV1 = S2AV1+(1.0/Y)*(SS1-S2AV1)
      IF( IDENT1(N) - IDENT2(N)) 1022, 1023, 1022
C
 1023 MAX=MAX-0
      DO 1024 M=1,MAX
      MP=M
      XDATA(M,2) = XDATA(MP,1)
 1024 CONTINUE
      GO TO 1026
C
 1022 SUM=0.0 $ SS2=0.0
      DO 1041 I = 1, MAX
      XDATA(I,2)=XDATA(I,2)
      XDATA(I,2)=XDATA(I,2)/(-409.6)
      SS2=SS2 + XDATA(I,2)**2
 1041 SUM = SUM + XDATA(I,2)
      XMEAN = SUM/XMAX
      SS2=SS2/(XMAX-1.0) $ SIGMA2= SQRTF(SS2)
      IF(NORM-1)1103,1102,1103
 1103 SIGMA2=1.0
 1102 PRINT 1060 $ PRINT 1061,IDENT2(N),XMEAN,SS2,SIGMA2
      DO 1025 I = 1,MAX
 1025 XDATA(I,2)=(XDATA(I,2)-XMEAN)/SIGMA2
 1026 CONTINUE
C     DATA NOW CONVERTED TO VOLTS AND MEAN REMOVED
C
```

```
C     FORM NOW THE AUTO/CROSS CORRELATION FUNCTION
      XX=N $ JINC=0
      DO 1015 L=1, NOSHIFTS
           SUM=0.0
           DO 1016 K=1,MAXTAU
      J=K+L-1+JINC
 1016          SUM=SUM + XDATA(K,2)*XDATA(J,1)
           RTAU(L) = SUM/XMAXTAU $ KSHIFTS(L) = L-1
      JINC = JINC + MISS
 1015 CONTINUE
C     NORMALIZE RTAU
      ARBAR=0.0 $ FIX=1.0/RTAU(1)
      DO 602 J=1,NOSHIFTS
      RTAU(J)=RTAU(J)*FIX
      RBAR(J)=RBAR(J)+(1.0/XX)*(RTAU(J)-RBAR(J))
  602 CONTINUE
 1006 CONTINUE
      MOD=0 $ LAB=4HSINE$ READ 1017,(ITITLE(I),I=1,6) $ NUMB=100
      READ 1017,(ITITLE(I),I=7,12)
      CALL DRAW(NUMB,KSHIFTS,RSS,MOD,0,LAB,ITITLE,0,0,4,0,2,2,6,8,0,LAST
     1)
      READ 1017,(ITITLE(I),I=1,6) $ READ 1017,(ITITLE(I),I=7,12)
      IDENT=NAMESIG $ M=NOSHIFTS-1
      CALL PSD(RSS,DELTAT,M,IDENT,ITITLE,IPRINT,IPICT,RBARPT)
      PRINT 1012 $ PRINT 2042 $ PUNCH 2042
 2042 FORMAT(48H POWER SPECTRUM PURE SIGNAL 50CPS, 5.0 CPS COMB /)
      M1=1 $ M2=6
      DO 708 J=1,21
      PRINT 701,(RBARPT(M),M=M1,M2) $ PUNCH 701,(RBARPT(M),M=M1,M2)
      M1=M2+1 $ M2=M2+6
  708 CONTINUE
      MOD=0 $ LABEL=4HNOIZ
      READ 1017,(ITITLE(I),I=1,6) $ READ 1017,(ITITLE(I),I=7,12)
      CALL DRAW(NUMB,KSHIFTS,RNN,MOD,0,LABEL,ITITLE,0,0,4,0,2,2,6,8,0,LA
     1ST)
      READ 1017,(ITITLE(I),I=1,6) $ READ 1017,(ITITLE(I),I=7,12)
```

```
      IDENT=NAMENSE $ M=NOSHIFTS-1
      CALL PSD(RNN,DELTAT,M,IDENT,ITITLE,IPRINT,IPICT,RBARPT)
      PRINT 1012 $ PRINT 2043 $ PUNCH 2043
 2043 FORMAT(45H    POWER SPECTRUM OF NOISE AVG OF 10 BLOCKS /)
      M1=1 $ M2=6
      DO 709 J=1,21
      PRINT 701,(RBARPT(M),M=M1,M2) $ PUNCH 701,(RBARPT(M),M=M1,M2)
      M1=M2+1 $ M2=M2+6
  709 CONTINUE
      MOD=0 $ LABEL=4H S+N
      READ 1017,(ITITLE(I),I=1,6) $ READ 1017,(ITITLE(I),I=7,12)
      CALL DRAW(NUMB,KSHIFTS,RBAR,MOD,0,LABEL,ITITLE,0,0,4,0,2,2,6,8,0,L
     1AST)
      PRINT 1012 $ PRINT 2041 $ PUNCH 2041
 2041 FORMAT(32H    RBAR AFTER 6 AVG. S/N =-10DB/)
      M1=1$ M2=6
      DO 707 J=1,21
      PRINT 701,(RBAR(M),M=M1,M2) $ PUNCH 701, (RBAR(M),M=M1,M2)
      M1=M2+1 $ M2=M2+6
  707 CONTINUE
      READ 1017,(ITITLE(I),I=1,6) $ READ 1017,(ITITLE(I),I=7,12)
      IDENT=IDENT1(6) $ M=NOSHIFTS-1
      CALL PSD(RBAR,DELTAT,M,IDENT,ITITLE,IPRINT,IPICT,RBARPT)
      M1=1 $ M2=6 $ PRINT 1012 $ PRINT 1044$ PUNCH 1044
 1044 FORMAT(21H S+N PSD AFTER 6 AVG./)
      DO 710 J=1,21
      PRINT 701,(RBARPT(M),M=M1,M2) $ PUNCH 701,(RBARPT(M),M=M1,M2)
      M1=M2+1 $ M2=M2+6
  710 CONTINUE
 9999 REWIND 1
      END
```

```
      SUBROUTINE PSD(A,DELTAT,M,IDENT,ITITLE,IPRINT,IPICT)
      DIMENSION A(500),X(500),FREQ(500),TAU(500),ITITLE(12)
C     CALLING ARGUMENTS FOR S/R PSD
C          A = AUTO(CORRELATION WITH A(1) AT TAU = 0
C     DELTAT = TIME SPACING BETWEEN CORR. SAMPLES IN S E C O N D S .
C          M = NUMBER OF CORRELATION SHIFTS
C      IDENT = OCTAL IDENTIFICATION OF DATA RECORD
C     ITITLE = TITLE OF PSD GRAPH IF DESIRED
C     IPRINT = FLAG SET TO 1 IF YOU WISH PRINT OUT PSD
C      IPICT = FLAG SET TO 1 IF YOU WANT TO PLOT PSD
C     IF BOTH IPRINT AND IPICT ARE ZERO  PROGRAM PRINTS PSD ANYWAY
C
  103 FORMAT(53H1      T U K E Y     S P E C T R U M     E S T I M A T E//)
  104 FORMAT(8H IDENT= O16,8H    M   = I4,9H DELTAT= F9.6 //)
  105 FORMAT(8H XFACT= F8.5,16H          A(0) = E12.5 //)
  106 FORMAT(53H   TAU(SEC)         R(TAU)        FREQ(CPS)       X(FREQ)      )
  107 FORMAT(1X,F11.7,3X,F10.5,3X,F9.3,3X,F10.5)
  108 FORMAT(8H  COMB= F9.5,15H    FMAX CPS = F12.3 //)
C
C     FIND  X(1)---THE POWER SPECTRAL DENSITY AT FREQ = 0.0 -----
   50 ASUM = 0.0 $ FM = M $ PI = 4.0*ATANF(1.0)
      CS1 = COSF(PI/FM) $ SN1 = SINF(PI/FM)
      CSL=CS1 $ SNL=SN1
C
      DO 52 L=2,M
      AZ=(1.0 + CSL) $ ASUM = ASUM + AZ*A(L)
      CSL1=CSL*CS1-SNL*SN1 $ SNL1=SNL*CS1+CSL*SN1
      CSL=CSL1 $ SNL=SNL1
   52 CONTINUE
C
      X(1) = 0.5*(ASUM + A(1))/FM
C     FIND X(K)---POWER SPECTRUM AT K=2,M
      CSK=CS1 $ SNK=SN1 $ MZ = M + 1
C
```

```
 53 DO 59 K=2,MZ
    ASUM=0.0 $ CSKL=CSK $ SNKL=SNK $ CSL=CS1 $ SNL=SN1
 54      DO 55 L=2,M
         AZ=(1.0+CSL)*CSKL $ ASUM=ASUM+AZ*A(L)
        CSL1=CSL*CS1-SNL*SN1 $ SNL1=SNL*CS1+CSL*SN1
         CSL=CSL1 $ SNL=SNL1
         CSKL1=CSKL*CSK-SNKL*SNK $ SNKL1=SNKL*CSK+CSKL*SNK
         CSKL=CSKL1 $ SNKL=SNKL1
 55      CONTINUE
    IF(K-MZ) 56,57,57
 56 DZ=1.0/FM $ GO TO 58
 57 DZ=0.5/FM $ GO TO 58
 58 X(K)=DZ*(ASUM+A(1))
    CSK1=CSK*CS1-SNK*SN1 $ SNK1=SNK*CS1+CSK*SN1
    CSK= CSK1 $ SNK=SNK1
 59 CONTINUE
C
C   X(K) IS THE POWER SPECTRAL DENSITY AT FREQ. 0.0 CPS TO FMAX CPS
C   APPLY TRAP. RULE TO FIND ENERGY CONTAINED IN POWER SPECTRUM
C   FOR RANGE OF K=1,M+1 - - - -  DEFINE ENERGY AS XENGY
C   NOTE-- X(K) IS ENERGY W.R.T. UNIT CHANGE OF INDEX K , NOT CPS ----
    ASUM=0.0
 65 DO 66 K=2,M
    ASUM=ASUM+X(K)
 66 CONTINUE
    XENGY=0.5*(X(1)+2.0*ASUM+X(M+1))
C   FIND FRACTION OF TOTAL ENERGY IN CALCULATED FREQ. RANGE
    XFACT=XENGY/A(1)
C   XFACT SHOULD = 1.0 IF ALL FREQUENCIES HAVE BEEN ACCOUNTED FOR...
C   OBTAIN SPECTRAL DENSITY W.R.T. CPS -- I.E., NORMALIZE W.R.T. XENGY
    FREQ(1)=0.0 $ TAU(1)=0.0 $ MZ=M+1
C
    AZ=2.0*DELTAT*FM $ COMB=1.0/AZ
```

```
67 DO 68 K=1,NZ
   X(K)=AZ*X(K)/XENGY $ FREQ(K+1)=FREQ(K)+COMB
   TAU(K+1)=TAU(K)+DELTAT
68 CONTINUE
C
C     POWER SPECTRUM WRITE-OUT INSTRUCTIONS----
   PRINT 103 $ PRINT 104,IDENT,M,DELTAT
   PRINT 105,XFACT,A(1)
   PRINT 108, COMB, FREQ(NZ)
   JTEST=0
   IF(IPRINT-1) 40,41,40
41 PRINT 106 $ PRINT 107 ,(TAU(K),A(K),FREQ(K),X(K),K=1,NZ) $ JTEST=1
40 IF(IPICT-1) 42,43,42
43 MOD=0 $ LABEL=4HFREQ $ JTEST=1
   DO 44 J=1,NZ
44 X(J)=X(J)*(-1.0)
C     THE SIGN INVERSION ON X(K) IS A CONVENIENCE FOR PLOTTING
   CALL DRAW(NZ,X,FREQ,MOD,0,LABEL,ITITLE,0.,0.,6,0,2,6,0,1,LAST)
42 IF(JTEST-1) 41,45,41
45 CONTINUE
   END
```

```
-COOP,,BARRETT N BOX B ,0/49/S/1S/2S/E/45=54,10,10000.
-FTN,L,E.
      PROGRAM PROCES
      DIMENSION RSS(150),RNN(150),RBAR(150),FREQ(150),KSHIFTS(150)
      DIMENSION PSDN(150),PSDSPN(150),PSDS(150),ENGY(3),IT(12),Z(150)
      DIMENSION ZM(150) , SIG(150)
      TYPE REAL KSHIFTS
  200 FORMAT(10X,6F10.7)
  201 FORMAT(1H1)
  203 FORMAT(23H ALPHA TOO BIG   AT AL=F10.7,3X,5H KNT=I2,3X,3H L=I3,3X,
     16H Z(L)=F10.7/)
  204 FORMAT(21H ALPHA CORRECT AT AL=F10.7,2X,8H ON TRY I2/)
  205 FORMAT(6A8)
  206 FORMAT(42H                  FREQ        PSD           R(TAU) /)
  207 FORMAT(11X,F8.2,2X,F10.7,3X,F10.7,/)
  208 FORMAT(54H             OUTPUT DETERMINED BY AL SEARCH IN PSD DOMAIN/)
      C=0.0 $ CALL TIME $ PRINT 201
C     ZERO-IZE ARRAYS
      DO 303 K=1,150
      RSS(K)=C$RNN(K)=C$RBAR(K)=C$FREQ(K)=C$KSHIFTS(K)=C$PSDN(K)=C
      PSDSPN(K)=C $ PSDS(K)=C $ Z(K)=C
  303 CONTINUE
C     READ IN PREFORMED CORRELATIONS AND POWER SPECTRAL DENSITIES.
      DO 300 L=1,6
      M1=1 $ M2=6
      DO 301 J=1,21
      GO TO (101,102,103,104,105,106) L
  101 READ 200,(RSS(M),M=M1,M2) $ GO TO 107
  102 READ 200,(PSDS(M),M=M1,M2)$ GO TO 107
  103 READ 200,(RNN(M),M=M1,M2) $ GO TO 107
  104 READ 200,(PSDN(M),M=M1,M2)$ GO TO 107
  105 READ 200,(RBAR(M),M=M1,M2)$GO TO 107
  106 READ 200,(PSDSPN(M),M=M1,M2) $ GO TO 107
  107 M1=M2+1 $ M2=M2+6
  301 CONTINUE
  300 CONTINUE
```

```
C     SET UP KSHIFTS AND FREQ.
      C=5.0 $ F=0.0
      DO 302 L=1,126
      KSHIFTS(L)=L-1 $ FREQ(L)=F $ F=F+C
  302 CONTINUE
C     -20DB OR WHATEVER DATA NOW IN AS A/C AND PSD
      CALL TIME
C     FIND ALPHA
C     PROGRAM  DISCONTINUES SEARCH WHEN AT LEAST ONE Z IS ZERO AND NO
C      POINTS ARE NEGATIVE .
      AP=1.0 $ AN=0.0 $ KNT=0 $ KEY=0
  405 AL=(AP+AN)/2.0
      DO 304 L=1,100
      Z(L)=PSDSPN(L)-AL*PSDN(L)
      IF(Z(L))400,401,402
  402 IF(Z(L)-.000001)401,401,304
  401 KEY=1
  304 CONTINUE
      IF(KEY-1)403,404,403
  403 AN=AL $ KNT=KNT+1
      IF(KNT-20)405,405,404
  400 AP=AL $ KNT=KNT+1$PRINT 203,AL,KNT,L,Z(L) $ GO TO 405
  404 PRINT 204,AL,KNT $ CALL TIME
      DO 310 L=1,126
  310 Z(L)=PSDSPN(L)-AL*PSDN(L)
C     COMPUTE PSD NORMALIZING FACTOR .
      A=Z(1)+Z(126) $ B=0.0
      DO 305 L=2,125
  305 B=B+Z(L)
      T=2.0/(A+4.0*B)
C     RENORMALIZE PSD AND USE AL TO FORM R(SS)=R(S+N)-AL$R(NN)  .
      DO 307 L=1,126
      Z(L)=Z(L)*T
      ZM(L)=Z(L)*(-1.0)
  307 SIG(L)=RBAR(L)-AL*RNN(L)
C      RENORMALIZE .
```

```
      FIX=1.0/SIG(1)
      DO 308 L=1,126
  308 SIG(L)=SIG(L)*FIX
C     DRAW COMPUTED PSD .
      MOD=0 $ LABEL=4HFREQ $ MZ=126
      READ 205,(IT(I),I=1,6) $ READ 205,(IT(I),I=7,12)
      CALL DRAW(MZ,ZM,FREQ,MOD,0,LABEL,IT,0,0,0,6,0,2,6,8,1,LAST)
      LABEL=4HBEST $ MP=100
      READ 205,(IT(I),I=1,6) $ READ 205,(IT(I),I=7,12)
C     DRAW COMPUTED SIGNAL CORRELATION
      CALL DRAW(MP,KSHIFTS,SIG,MOD,0,LABEL,IT,0,0,4,0,2,2,6,8,0,LAST)
C     PRINT RESULTS .
      PRINT 201 $ PRINT 208 $ PRINT 206
      DO 309 J=1,126
  309 PRINT 207,FREQ(J),Z(J),SIG(J)
      END
```

```
               IDENT        TIME
               ENTRY        TIME
     TIME      SLJ          **
               ENQ          0
               LAC     7    =00
               SCL          =04000000000000000
               DVI          =074
               STQ          =SIT3
               ENQ          0
               DVI          =074
               STA          =SIT1
               STQ          =SIT2
               CALL         TIMO
    +          ZRO          IT1
               ZRO          IT2
               ZRO          IT3
    +          SLJ          TIME
               END
    C
             SUBROUTINE TIMO (IT1,IT2,IT3)
           3 FORMAT(26H0 INITIAL TIME IN SECONDS ,F10.4          )
           5 FORMAT(28H0 TIME DIFFERENCE IN SECONDS,F10.4       )
             DECT=FLOATF(IT1)*60.+FLOATF(IT2)+FLOATF(IT3)/60.$IF(XX-37.)1,2,1
           1 XX=37. $DECTO=0.0$PRINT 3,DECTO$DECTO=DECT$GO TO 4
           2 DECTO=DECT-DECTO$PRINT 5,DECTO $DECTO=DECT
           4 CONTINUE
             END
```

INPUT DATA TO PROGRAM PROCES. PRE-FORMED IN PROGRAM SIMSIG.

AUTO-CORRELATION OF PURE SIGNAL . 3750 SAMPLES.
READ ACROSS IN STEPS OF 800 MICRO-SECONDS .

1.0000000	.9703560	.8827537	.7423788	.5576656	.3396452	)
.1013469	-.1429989	-.3788161	-.5919907	-.7698039	-.9016175	)
-.9795789	-.9989920	-.9586836	-.8610925	-.7120446	-.5204734	)
-.2978154	-.0573786	.1864597	.4191692	.6268009	.7970022	)
.9195814	.9872423	.9959028	.9450060	.8376641	.6802613	)
.4822745	.2554968	.0134950	-.2292871	-.4583656	-.6600545	)
-.8223332	-.9354786	-.9927530	-.9907048	-.9294515	-.8126863	)
-.6473912	-.4434669	-.2130604	.0300299	.2713397	.4964288	)
.6918499	.8459815	.9495723	.9964777	.9838288	.9123987	)
.7864624	.6135589	.4040465	.1704057	-.0733777	-.3127909	)
-.5335205	-.7223993	-.8681459	-.9620670	-.9985338	-.9753269	)
-.8938531	-.7589802	-.5787856	-.3640293	-.1275217	.1165749	)
.3537583	.5698084	.7518590	.8890227	.9730928	.9990471	)
.9652588	.8738138	.7301386	.5428640	.3231633	.0841582	)
-.1598803	-.3944032	-.6053693	-.7802006	-.9084382	-.9824302	)
-.9977110	-.9533539	-.8520249	-.6997760	-.5057216	-.2814317	)
-.0403295	.2031761	.4345762	.6400267	.8072670	.9262709	)
.9899594	.9944754	.9395139	.8284065	.6677629	.4672639	)
.2388325	-.0038422	-.2463019	-.4740523	-.6734821	-.8326894	)
-.9421364	-.9952926	-.9889334	-.9234732	-.8028242	-.6342147	)
-.4277261	-.1956824	.0480198	.2888652	.5124372	.7053726	)

INPUT DATA TO PROGRAM PROCES. PRE-FORMED IN PROGRAM SIMSIG.

POWER SPECTRAL DENSITY OF PURE SIGNAL , 3750 SAMPLES .
READ ACROSS IN STEPS OF 5 CPS., FROM ZERO FREQUENCY.

.0000318	-.0000380	.0000782	-.0000703	.0001478	-.0002088	)
.0004872	-.0012260	.0063175	.0686631	.0959554	.0319243	)
-.0024413	.0007260	-.0003274	.0001471	-.0001036	.0000510	)
-.0000462	.0000234	-.0000238	.0000114	-.0000152	.0000060	)
-.0000101	.0000032	-.0000071	.0000020	-.0000034	.0000048	)
-.0000024	-.0000002	-.0000037	-.0000002	-.0000029	-.0000002	)
-.0000019	.0000002	-.0000016	-.0000002	-.0000017	-.0000005	)
-.0000014	-.0000004	-.0000012	-.0000004	-.0000011	-.0000002	)
-.0000004	.0000000	-.0000010	-.0000006	-.0000009	-.0000005	)
-.0000009	-.0000005	-.0000008	-.0000004	-.0000006	-.0000004	)
-.0000006	-.0000004	-.0000007	-.0000004	-.0000006	-.0000003	)
-.0000005	-.0000002	-.0000002	-.0000002	-.0000004	-.0000003	)
-.0000004	-.0000003	-.0000004	-.0000003	-.0000004	-.0000003	)
-.0000004	-.0000002	-.0000003	-.0000002	-.0000003	-.0000003	)
-.0000004	-.0000002	-.0000002	-.0000002	-.0000003	-.0000002	)
-.0000002	-.0000002	-.0000002	-.0000002	-.0000003	-.0000002	)
-.0000002	-.0000001	-.0000002	-.0000003	-.0000003	-.0000002	)
-.0000003	-.0000003	-.0000003	-.0000003	-.0000001	.0000003	)
.0000002	-.0000001	-.0000003	-.0000002	-.0000002	-.0000002	)
-.0000001	-.0000002	-.0000001	-.0000000	-.0000001	-.0000001	)
-.0000002	-.0000002	-.0000002	-.0000002	-.0000002	-.0000001	)

INPUT DATA TO PROGRAM PROCES. PRE-FORMED IN PROGRAM SIMSIG.

AUTO-CORRELATION OF NOISE . 37500 SAMPLES .
READ ACROSS IN STEPS OF 800 MICRO-SECONDS .

1.0000000	.3968294	-.1792378	-.1185634	.0346723	.0309137	)
-.0038283	.0011826	.0063094	.0006076	.0050879	.0043928	)
-.0028681	-.0009986	-.0005560	-.0000518	-.0006613	-.0005114	)
.0038456	.0053433	.0001990	.0013698	.0058643	-.0020326	)
-.0116694	-.0000437	.0143975	.0050918	-.0111755	-.0039668	)
.0041339	-.0112853	-.0161378	.0005273	-.0031768	-.0218976	)
-.0101186	.0065994	.0051737	.0024567	.0037124	.0086762	)
.0078568	.0031051	-.0014067	-.0039805	.0017231	.0105014	)
.0028867	-.0060322	-.0054604	.0012556	-.0035138	-.0106700	)
-.0046901	.0086611	.0018275	-.0056384	-.0044821	.0004739	)
.0033102	-.0074441	-.0142864	-.0055511	.0123390	.0128324	)
.0025722	-.0050786	-.0007864	.0030723	-.0072937	-.0043776	)
.0045527	-.0005662	-.0079292	-.0090249	-.0051144	-.0050326	)
-.0075289	-.0021372	-.0037414	-.0094313	-.0069693	-.0052810	)
-.0111708	-.0200754	-.0158357	.0041032	.0083365	-.0032238	)
-.0047423	.0014430	.0003431	-.0115304	-.0150510	-.0001074	)
.0176517	.0213504	.0125497	.0005253	-.0072540	-.0045798	)
.0081293	.0097129	.0051911	.0031286	.0071851	.0020314	)
-.0117145	-.0055225	.0054954	.0009242	-.0083639	.0024055	)
.0191900	.0058598	-.0148196	-.0066916	.0113706	.0076088	)
.0000428	.0059114	.0062035	.0045864	-.0060193	-.0212387	)

INPUT DATA TO PROGRAM PROCES. PRE-FORMED IN PROGRAM SIMSIG.

POWER SPECTRAL DENSITY OF NOISE. 37500 SAMPLES .
READ ACROSS IN STEPS OF 5 CPS., FROM ZERO FREQUENCY.

.0009951	.0021510	.0022637	.0021408	.0022179	.0022701	)
.0020828	.0020679	.0020073	.0019056	.0021381	.0023524	)
.0023329	.0021951	.0020395	.0020020	.0020514	.0021779	)
.0023195	.0023589	.0022655	.0021725	.0022424	.0023151	)
.0023203	.0024762	.0026275	.0025970	.0025523	.0024366	)
.0025295	.0027554	.0025921	.0025255	.0026328	.0026543	)
.0027497	.0028227	.0029129	.0029132	.0027790	.0028161	)
.0028770	.0028224	.0027616	.0028764	.0030539	.0030416	)
.0029710	.0029515	.0028630	.0028273	.0027620	.0027791	)
.0029493	.0027655	.0024853	.0025999	.0027925	.0027967	)
.0025162	.0022271	.0021744	.0021318	.0019887	.0020764	)
.0022322	.0021805	.0021348	.0019460	.0016050	.0013787	)
.0013802	.0014538	.0014490	.0013925	.0012984	.0011707	)
.0010337	.0009413	.0008900	.0008548	.0008352	.0007916	)
.0006975	.0006184	.0005843	.0005625	.0005400	.0005262	)
.0005363	.0005311	.0004546	.0003958	.0004134	.0004198	)
.0003941	.0003763	.0003461	.0003019	.0002968	.0002986	)
.0002730	.0002555	.0002469	.0002335	.0002060	.0001935	)
.0002077	.0002106	.0001944	.0001796	.0001844	.0001855	)
.0001678	.0001553	.0001668	.0001849	.0001743	.0001510	)
.0001463	.0001533	.0001585	.0001524	.0001409	.0000683	)

INPUT DATA TO PROGRAM PROCES. PRE-FORMED IN PROGRAM SIMSIG.

AUTO-CORRELATION OF SIGNAL PLUS NOISE. 22500 SAMPLES .
READ ACROSS IN STEPS OF 800 MICRO-SECONDS .

1.0000000	.4439173	-.0769963	-.0215226	.1048754	.0780833	)
.0135810	-.0063845	-.0239472	-.0622569	-.0951673	-.0934803	)
-.0818881	-.0995029	-.1146868	-.0916602	-.0636853	-.0417132	)
-.0197400	.0081741	.0260239	.0327858	.0521986	.0787574	)
.0980710	.1123382	.1194213	.1064591	.0865406	.0667432	)
.0424790	.0250769	.0161526	-.0127813	-.0571357	-.0673204	)
-.0586578	-.0733818	-.0970769	-.1040049	-.0966984	-.0766545	)
-.0537504	-.0463495	-.0341914	-.0058542	.0227589	.0460851	)
.0671499	.0841308	.0885379	.0817629	.0769611	.0876891	)
.0998926	.0884941	.0527789	.0172207	-.0054047	-.0288318	)
-.0542848	-.0693802	-.0885979	-.1120211	-.1233454	-.1042367	)
-.0755449	-.0720369	-.0607090	-.0325541	-.0058853	.0156225	)
.0364612	.0492823	.0588412	.0736955	.0848619	.1013245	)
.1064140	.0854102	.0711593	.0636135	.0350632	-.0017402	)
-.0108436	-.0203113	-.0588061	-.0957885	-.1039716	-.0906704	)
-.0785147	-.0955570	-.1033738	-.0794742	-.0546282	-.0341931	)
-.0033095	.0354630	.0561675	.0764141	.0731144	.0757725	)
.0866899	.1003737	.1151346	.1064556	.0803289	.0475494	)
.0263095	.0093115	-.0188108	-.0493309	-.0684905	-.0611957	)
-.0897179	-.0970245	-.1100503	-.1122958	-.0920561	-.0664537	)
-.0472031	-.0296917	.0054417	.0378704	.0548960	.0817585	)

INPUT DATA TO PROGRAM PROCES. PRE-FORMED IN PROGRAM SIMSIG.

POWER SPECTRAL DENSITY OF SIGNAL PLUS NOISE. 22500 SAMPLES.
READ ACROSS IN STEPS OF 5 CPS., FROM ZERO FREQUENCY.

.0011419	.0022504	.0021724	.0019168	.0017912	.0017972	)
.0019177	.0019075	.0028142	.0090495	.0115456	.0050066	)
.0017452	.0021952	.0020649	.0018739	.0016241	.0018234	)
.0021097	.0019235	.0016744	.0018636	.0019774	.0017505	)
.0015717	.0018458	.0021645	.0022551	.0023877	.0023111	)
.0022367	.0023104	.0022264	.0021090	.0019257	.0021157	)
.0024880	.0025100	.0025621	.0025628	.0027442	.0031584	)
.0029437	.0023871	.0022985	.0025889	.0027437	.0026990	)
.0026393	.0025764	.0024703	.0022036	.0022420	.0025138	)
.0025456	.0026252	.0025425	.0022209	.0022022	.0023629	)
.0025050	.0025471	.0024303	.0022064	.0023129	.0021421	)
.0018168	.0017879	.0017939	.0016664	.0014858	.0013222	)
.0011950	.0011136	.0010738	.0011008	.0012050	.0011950	)
.0010189	.0008940	.0008424	.0007476	.0007505	.0008204	)
.0007455	.0006494	.0006264	.0005632	.0005171	.0005062	)
.0004827	.0004450	.0003868	.0003657	.0003793	.0003583	)
.0003395	.0003376	.0003327	.0003281	.0003108	.0002883	)
.0002889	.0002783	.0002398	.0002318	.0002426	.0002223	)
.0001913	.0001796	.0001828	.0001766	.0001731	.0001861	)
.0001850	.0001709	.0001643	.0001619	.0001574	.0001504	)
.0001413	.0001388	.0001410	.0001338	.0001307	.0000673	)

OUTPUT DATA FROM PROGRAM PROCESS. NOISE REMOVAL ACCOMPLISHED.

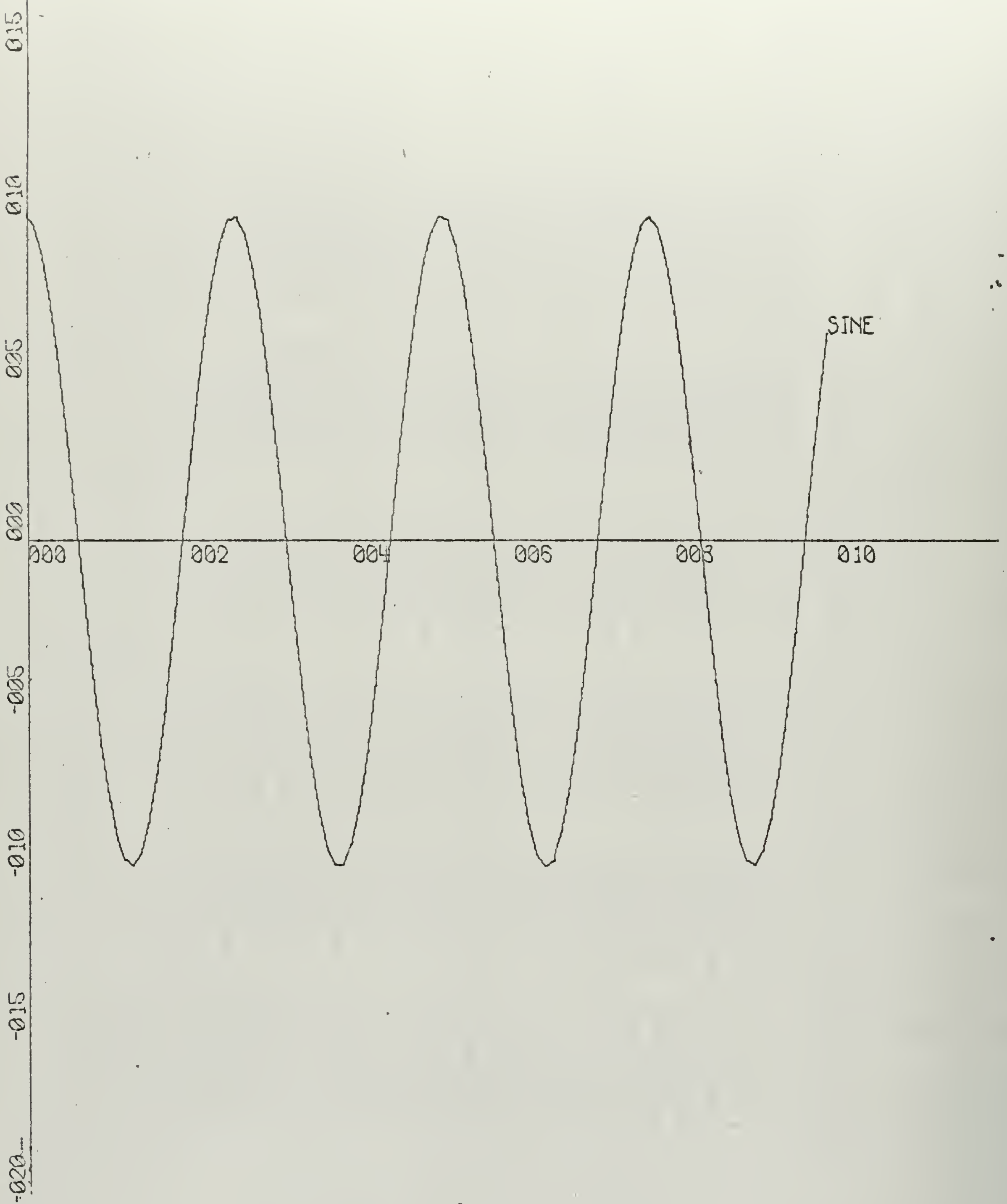
AUTO-CORRELATION OF SIGNAL PLUS NOISE. 22500 SAMPLES .
READ ACROSS IN STEPS OF 800 MICRO-SECONDS .

+1.0000000	.5427235	.1375407	.1821016	.2521851	.1770610
.0501116	-.0222628	-.0874357	-.1941677	-.3055364	-.2988509
-.2476986	-.3061980	-.3541715	-.2838854	-.1959308	-.1281684
-.0692305	.0141141	.0802132	.0987071	.1494235	.2482820
.3283432	.3481532	.3397965	.3191617	.2915818	.2151165
.1229399	.1013770	.0839087	-.0407072	-.1703596	-.1626327
-.1605093	-.2412092	-.3116330	-.3273971	-.3073940	-.2557070
-.1830230	-.1501219	-.1029847	-.0097859	.0668991	.1207517
.2019957	.2733231	.2857779	.2506943	.2458245	.2940795
.3193422	.2560105	.1596920	.0651868	-.0073406	-.0903250
-.1751385	-.1993429	-.2445284	-.3354309	-.4080568	-.3498870
-.2394608	-.2104396	-.1864467	-.1073103	-.0029316	.0575894
.1034159	.1538812	.1989479	.2472707	.2736624	.3244974
.3455045	.2691140	.2283261	.2169860	.1232615	.0056896
-.0101570	-.0208063	-.1489724	-.3053948	-.3396317	-.2741627
-.2333140	-.2990956	-.3210067	-.2220431	-.1376745	-.1057163
-.0472931	.0650760	.1786757	.2356543	.2417543	.2443786
.2515364	.2906105	.3458330	.3294670	.2338092	.1430615
.1060966	.0419874	-.0698134	-.1547830	-.1946563	-.2566191
-.3182433	-.3129103	-.3098763	-.3338889	-.3090801	-.2218772
-.1463409	-.1043990	.0038432	.1077114	.1827168	.2978812

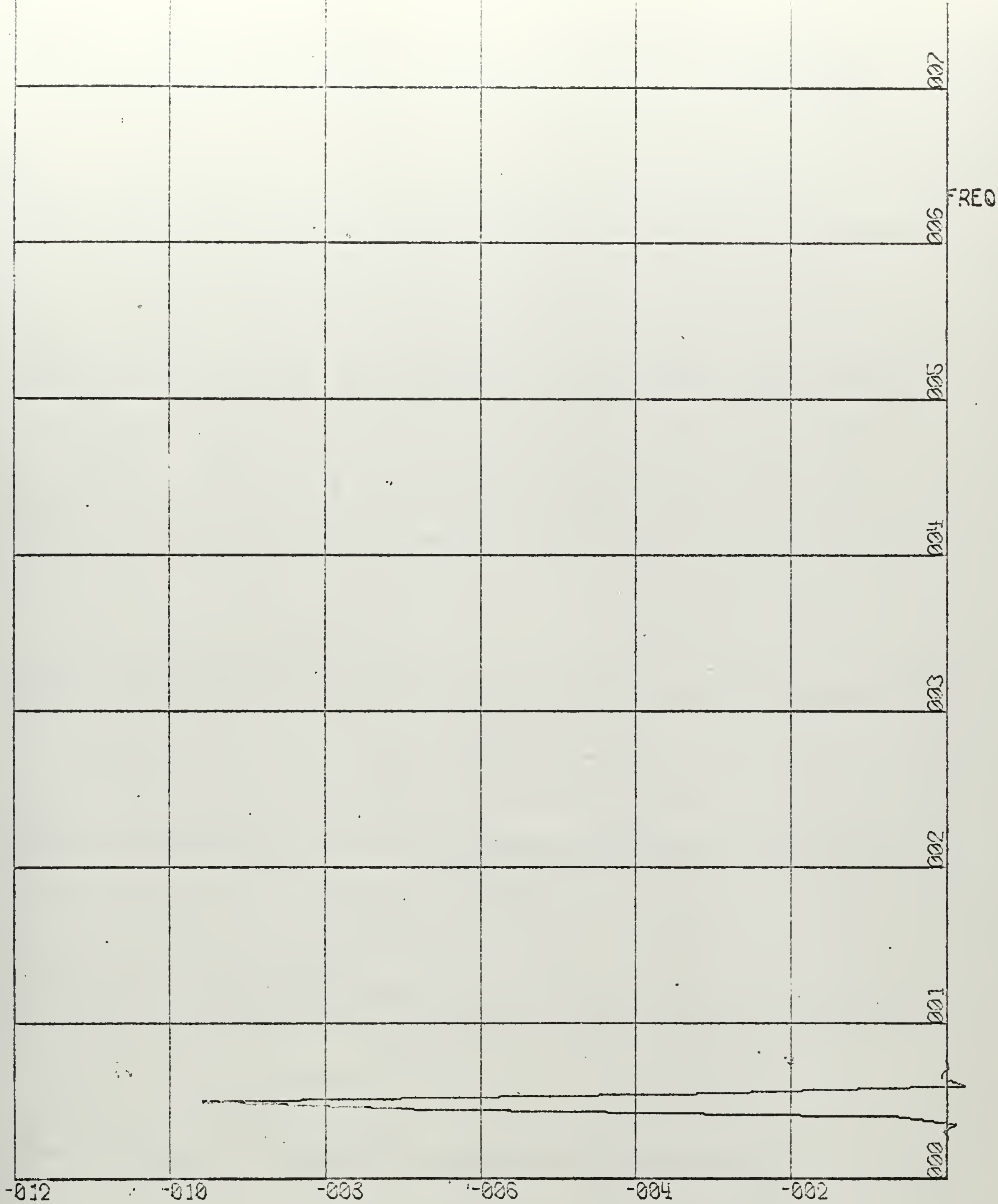
OUTPUT DATA FROM PROGRAM PROCESS. NOISE REMOVAL ACCOMPLISHED.

POWER SPECTRAL DENSITY OF SIGNAL PLUS NOISE. 22500 SAMPLES.
READ ACROSS IN STEPS OF 5 CPS., FROM ZERO FREQUENCY.

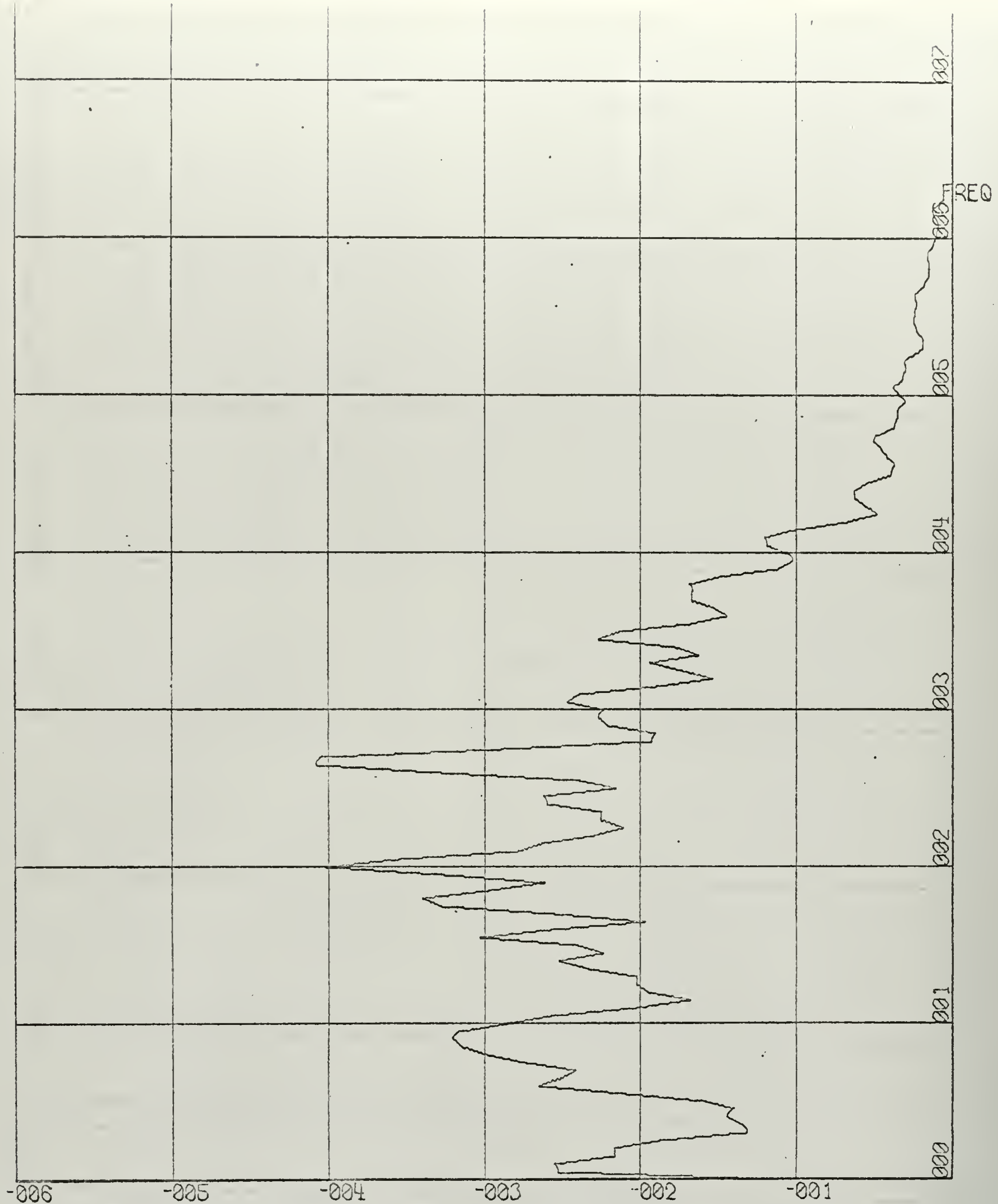
.0036317	.0061590	.0049614	.0036238	.0022438	.0020160
.0039356	.0039347	.0112897	.0602130	.0783620	.0264899
.0012824	.0054989	.0053055	.0040203	.0018221	.0027039
.0041816	.0025295	.0010872	.0030443	.0035601	.0014171
.0000022	.0013100	.0029881	.0038515	.0051154	.0051291
.0040634	.0034481	.0036545	.0030934	.0011070	.0024665
.0048563	.0046424	.0045736	.0045775	.0066906	.0097100
.0077237	.0036912	.0032181	.0049734	.0051893	.0049596
.0048674	.0044817	.0041235	.0022414	.0028826	.0040020
.0042543	.0058380	.0056689	.0035708	.0024134	.0036385
.0062155	.0088377	.0074322	.0059185	.0074971	.0057107
.0023673	.0024148	.0027015	.0027044	.0030950	.0030145
.0020198	.0010013	.0007176	.0012241	.0025273	.0031209
.0024743	.0019906	.0018596	.0013091	.0014346	.0022062
.0021195	.0017695	.0017902	.0014531	.0011743	.0011628
.0009273	.0006621	.0006125	.0007578	.0007708	.0005742
.0005634	.0006422	.0007629	.0009595	.0008520	.0006680
.0008072	.0008169	.0005633	.0005717	.0006000	.0007082
.0003930	.0002869	.0003969	.0004266	.0003742	.0004693
.0005538	.0005100	.0003084	.0002846	.0003054	.0003736
.0003276	.0002714	.0002612	.0002374	.0002738	.0001633



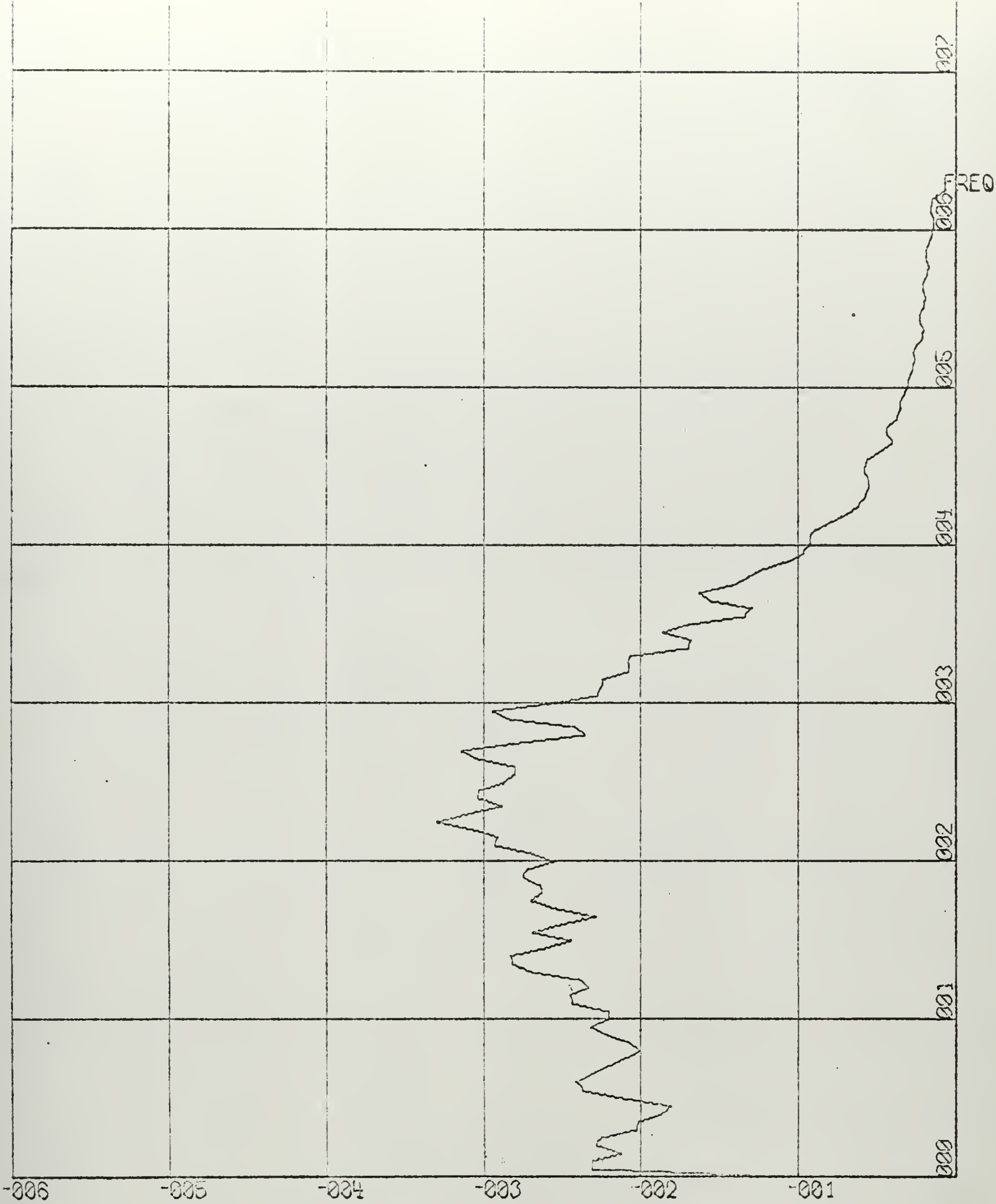
X Scale- 16 msec/inch. Y Scale- 0.5 units/inch.
Figure 4- Pure signal auto-correlation over 3750 samples. Estimated S/N ratio, plus 55 DB.



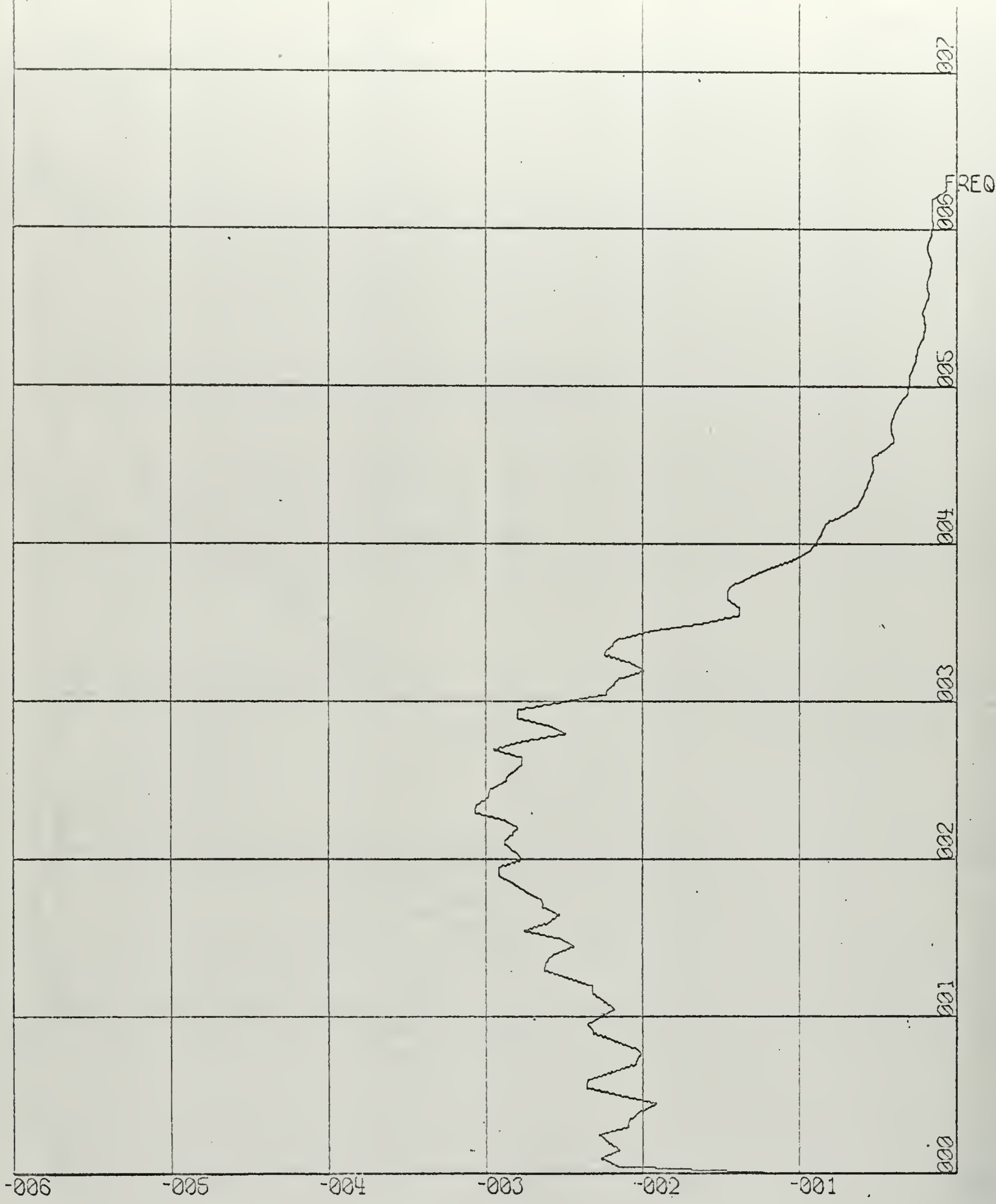
X Scale- 0.02 units/inch.   Y Scale- 100 cps/inch.
Figure 5- Pure signal power spectral density over 3750 samples. 125 estimates 5 cps apart.



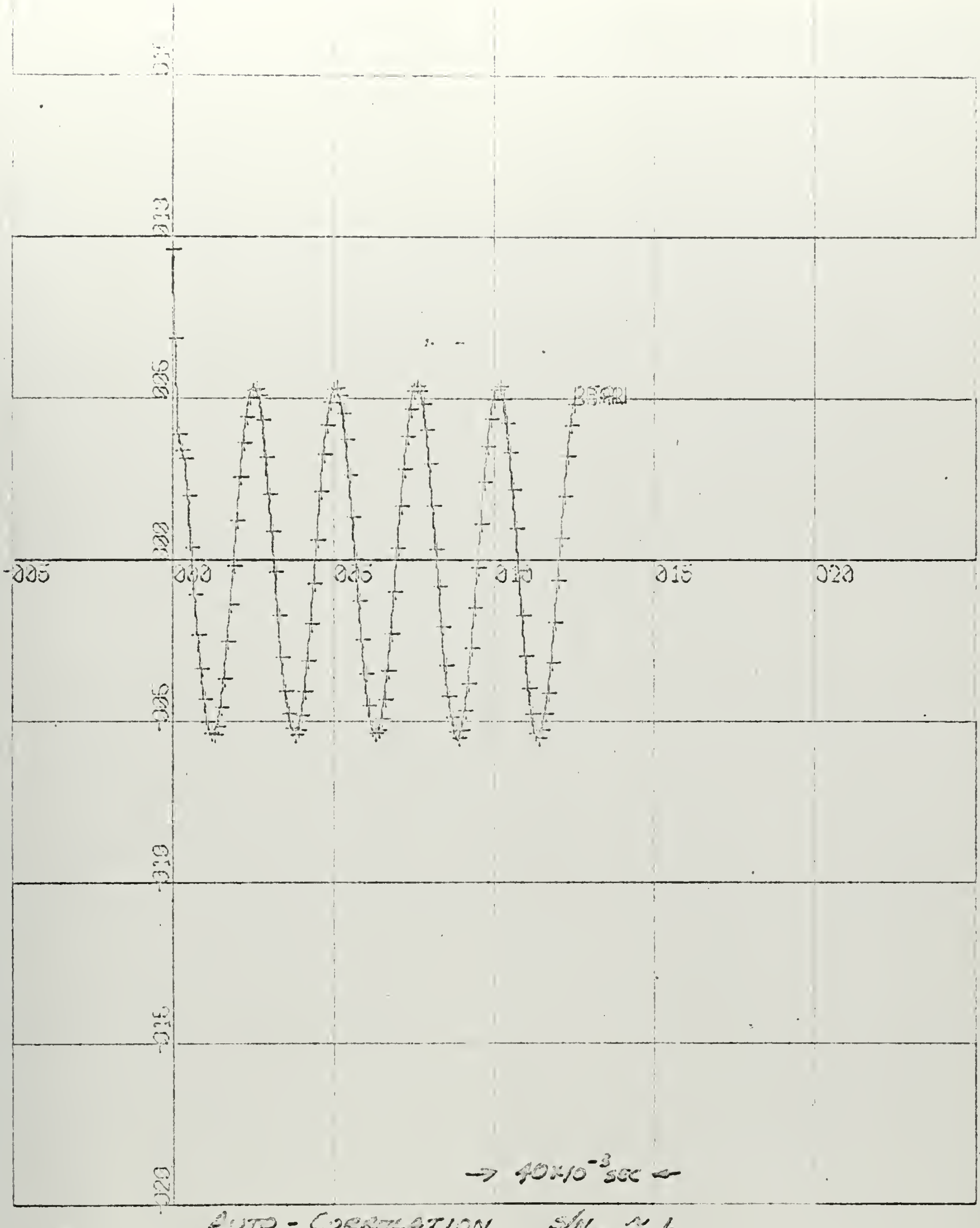
X Scale- .001 units/inch. Y Scale- 100 cps/inch.
Figure 6- Power spectral density of noise, 3750 samples, 125 estimates 5 cps apart.



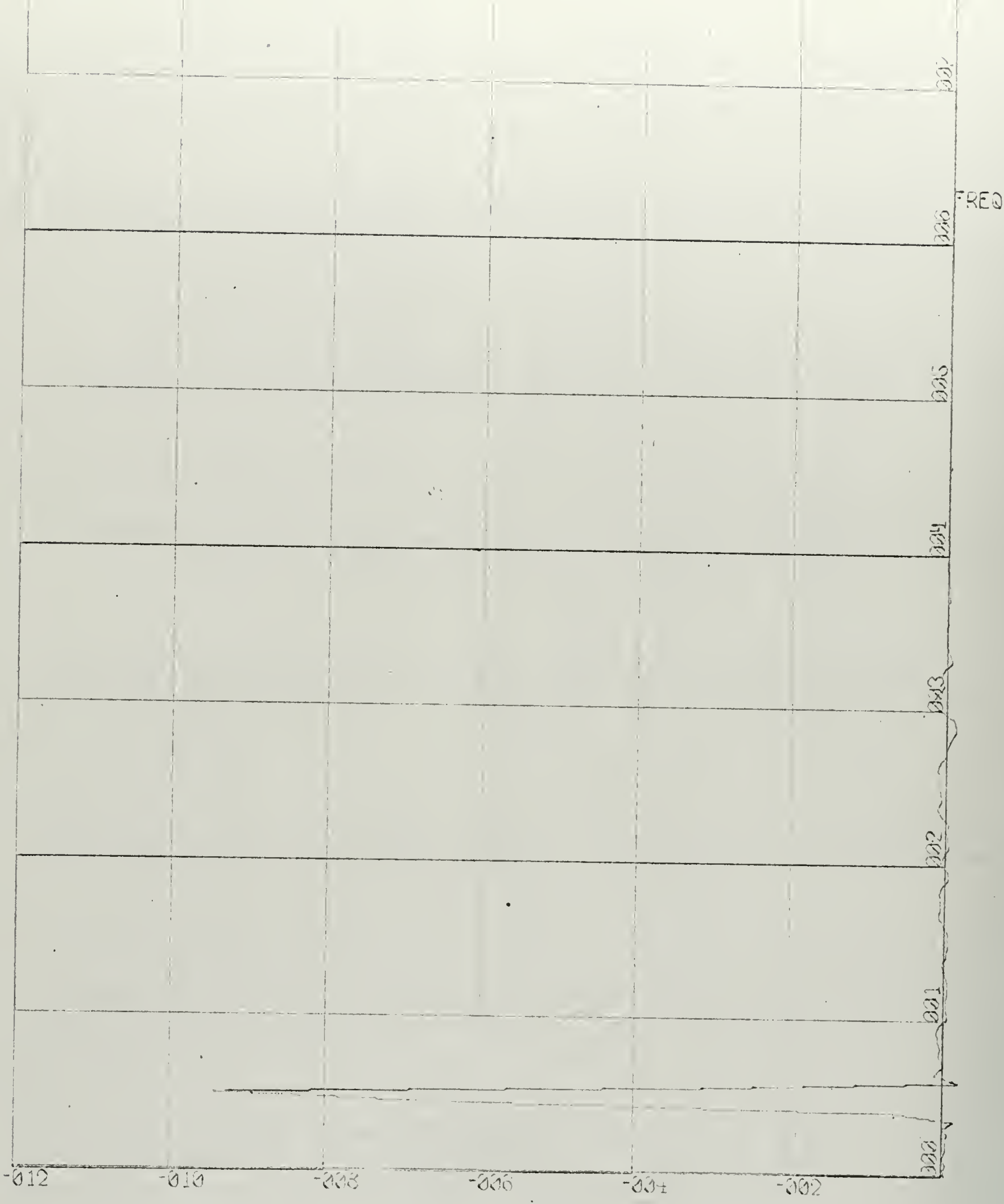
X Scale- .001 units/inch. Y Scale- 100 cps/inch
Figure 7- Power spectral density of noise,18750 samples,
125 estimates 5 cps apart.



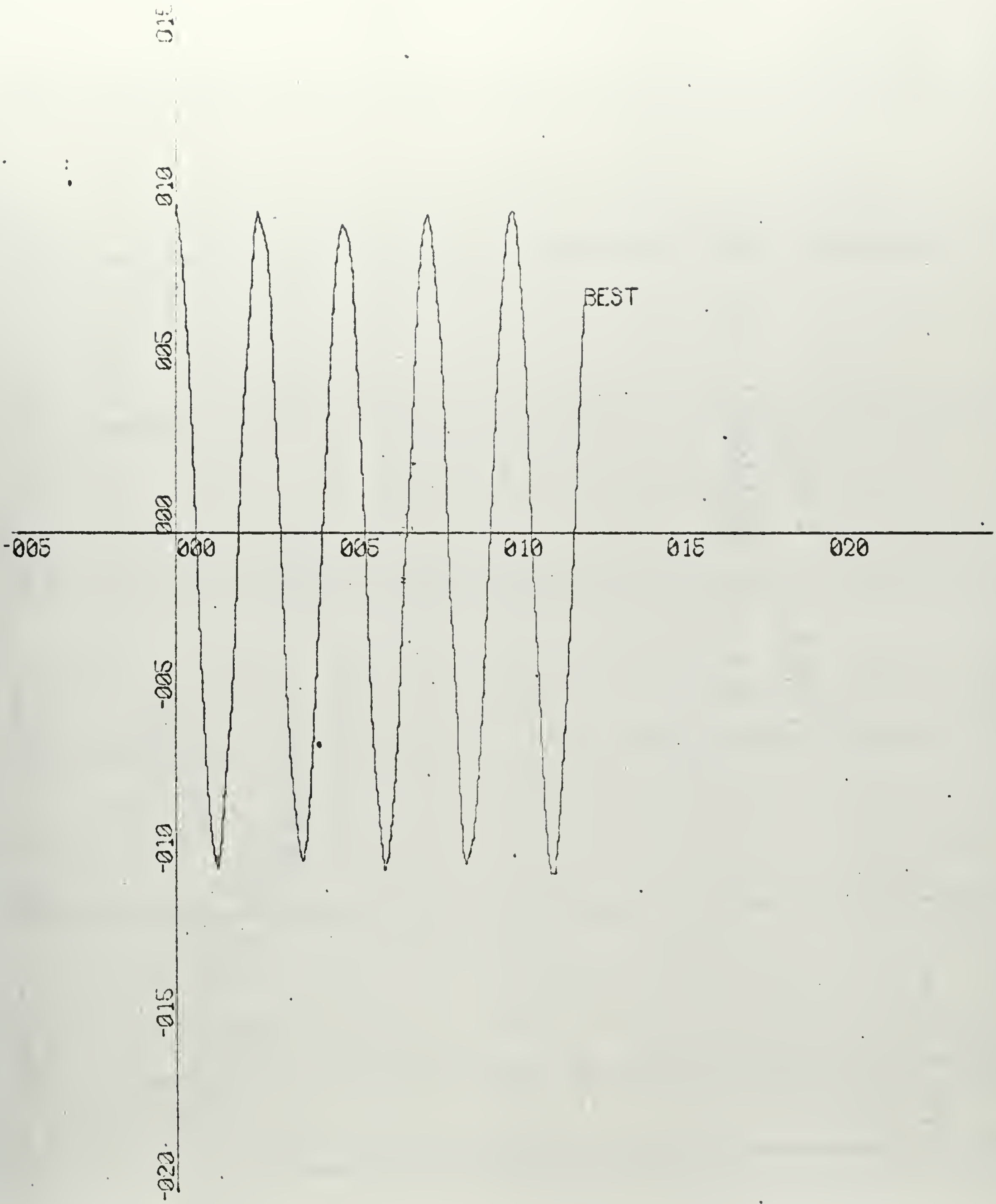
X Scale- .001 units/inch. Y Scale- 100 cps/inch.
Figure 8- Power spectral density of noise, 37500 samples, 125 estimates 5 cps apart.



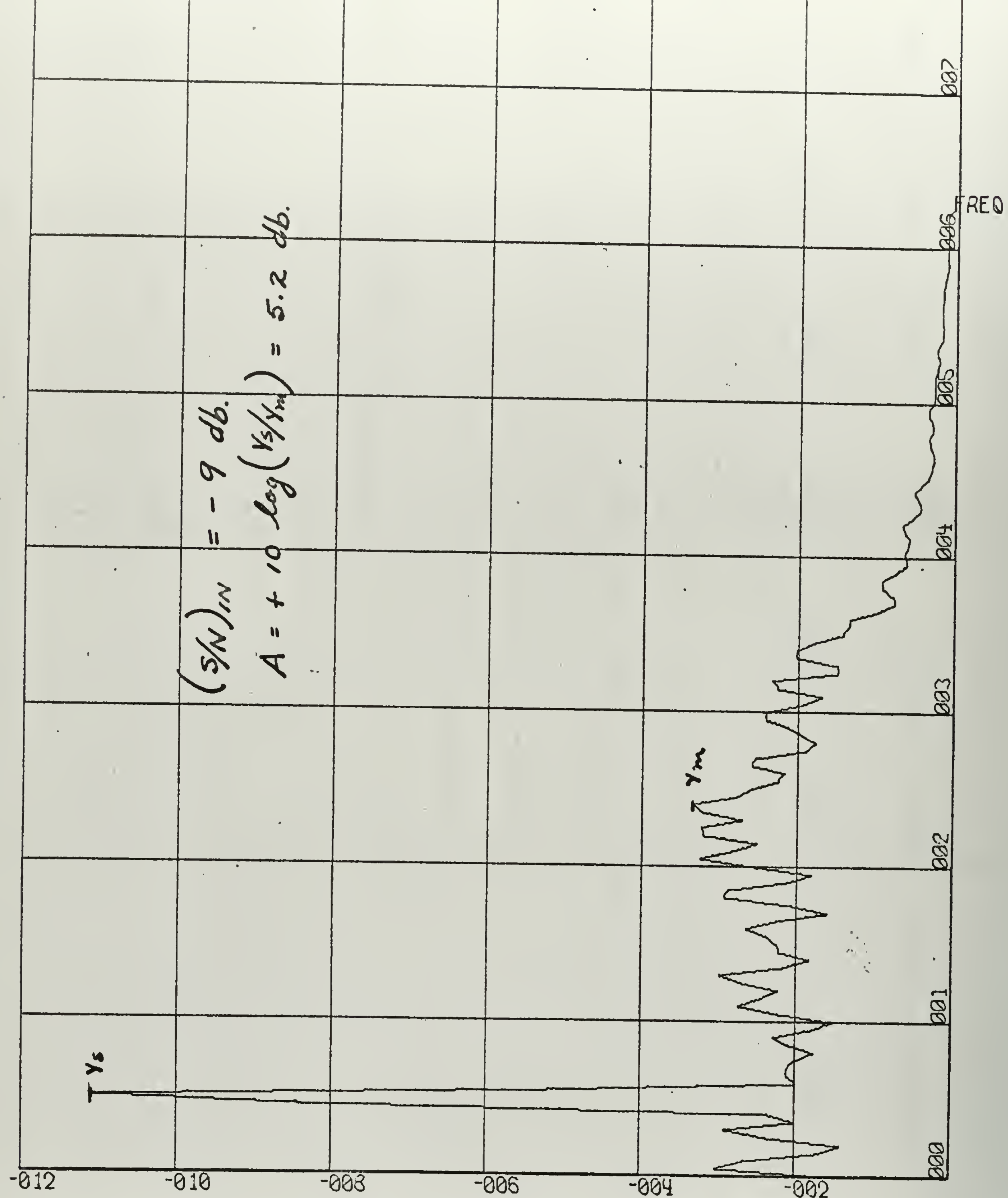
X Scale- 40 msec./inch. Y Scale- 0.5 units/inch.
Figure 9- Auto-correlation of ODB. signal plus noise.
3750 samples .



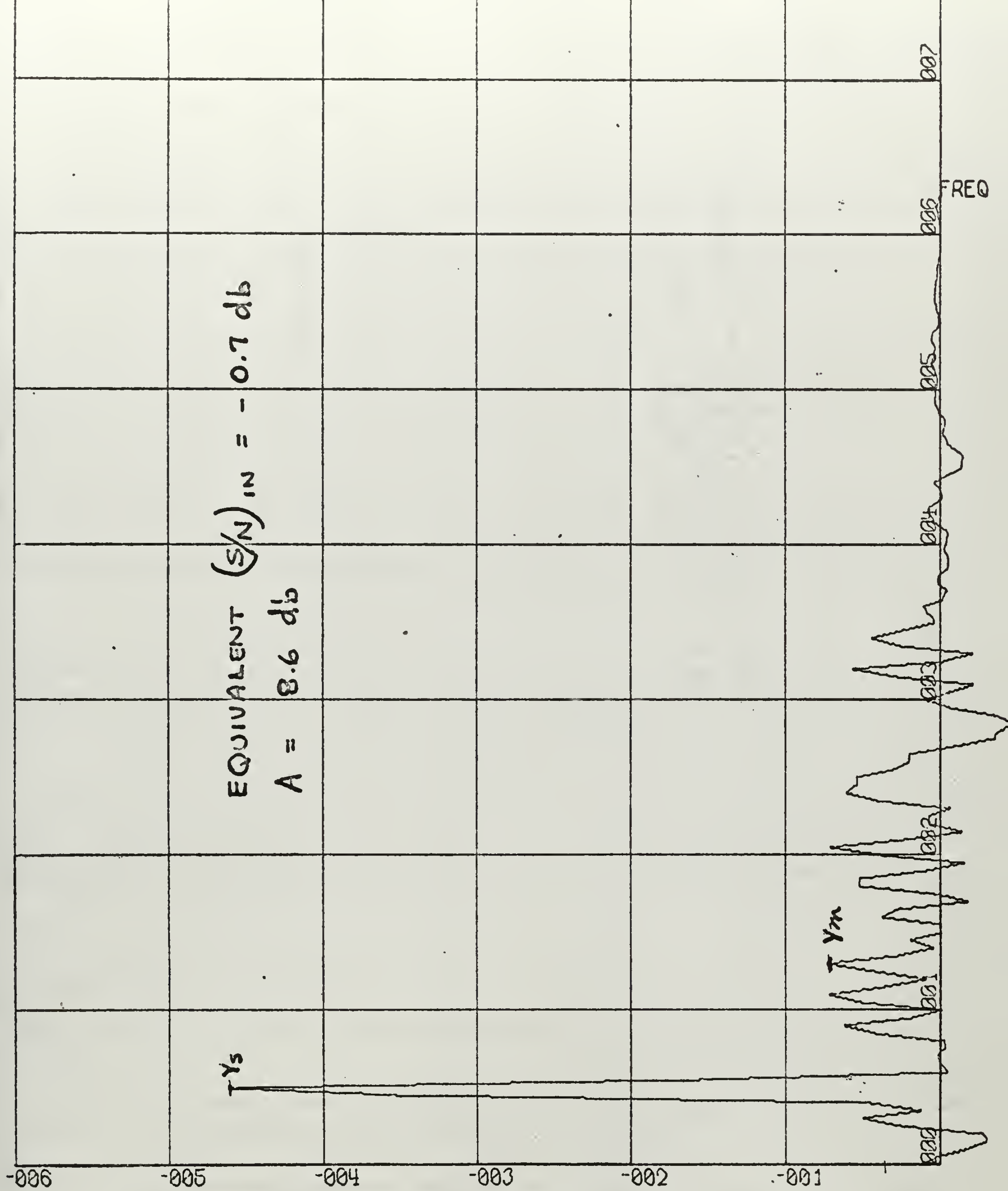
X Scale- 0.02 units/inch. Y Scale- 100 cps/inch.
Figure 10- Power spectral density of ODB. signal after noise removal. Alpha calculated on RBAR(T). 3750 samples .



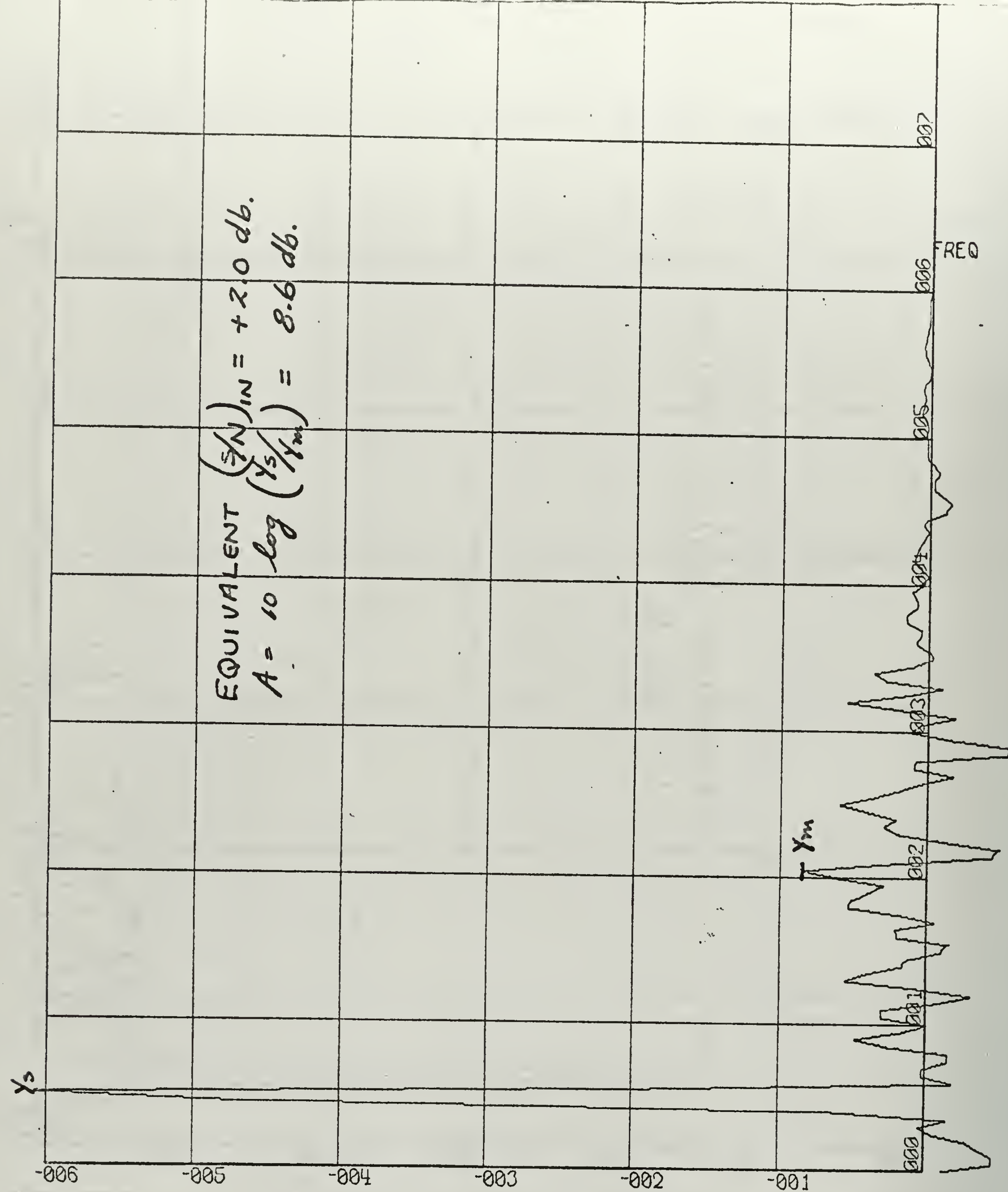
X-Scale- 40 msec./inch. Y Scale- .5 units/inch.
Figure 11- ODB. signal after noise removed. Alpha determined on RBAR(T). 3750 samples .



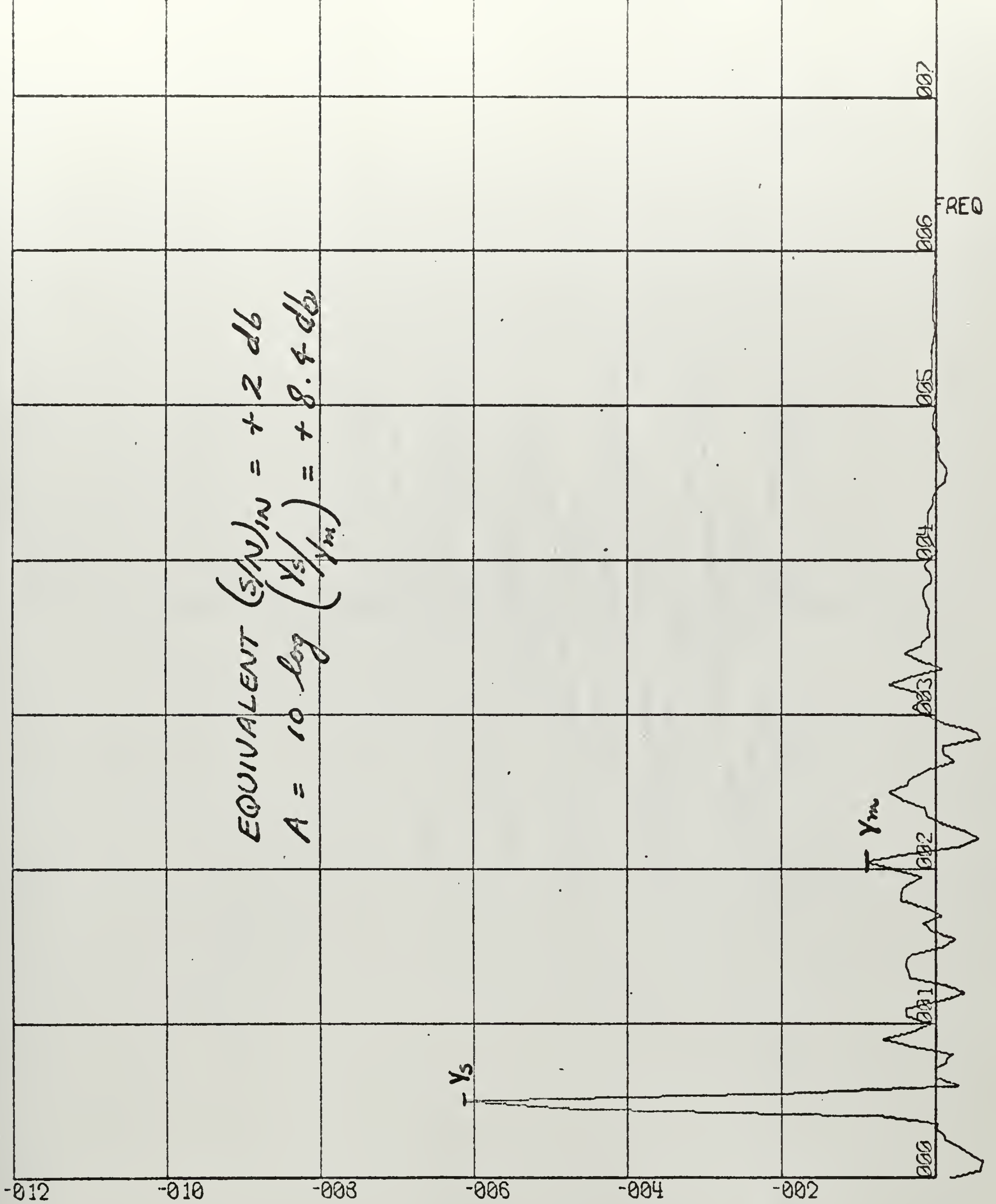
X Scale- .002 units/inch. Y Scale- 100 cps/inch.
Figure 12- -10DB. psd. before noise removal.
125 estimates, 5 cps apart, over 3750 samples.



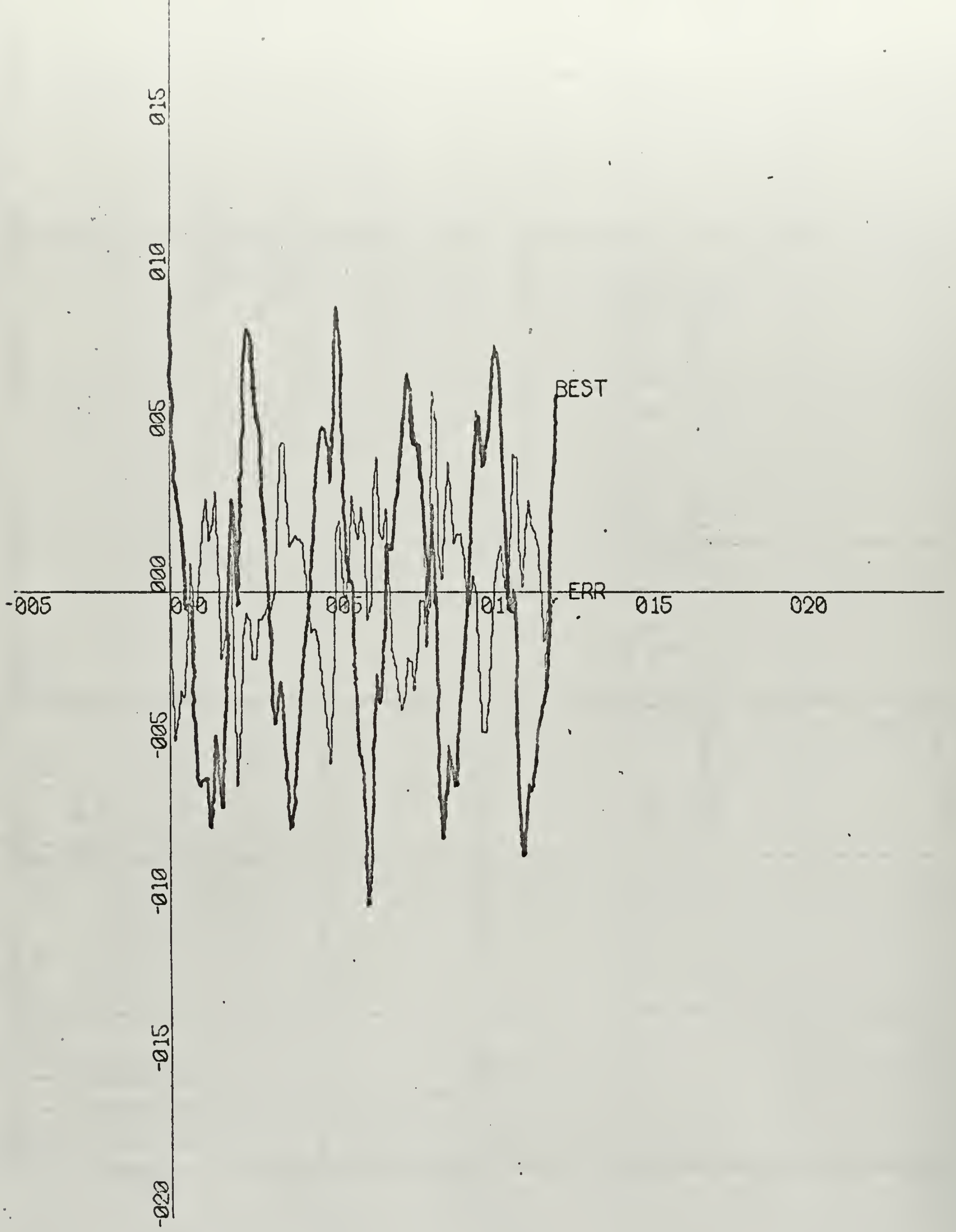
X Scale- .010 units/inch. Y Scale- 100 cps/inch.
Figure 13- -10DB. psd. after noise removed. 3750 samples.
Alpha determined on RBAR(T).



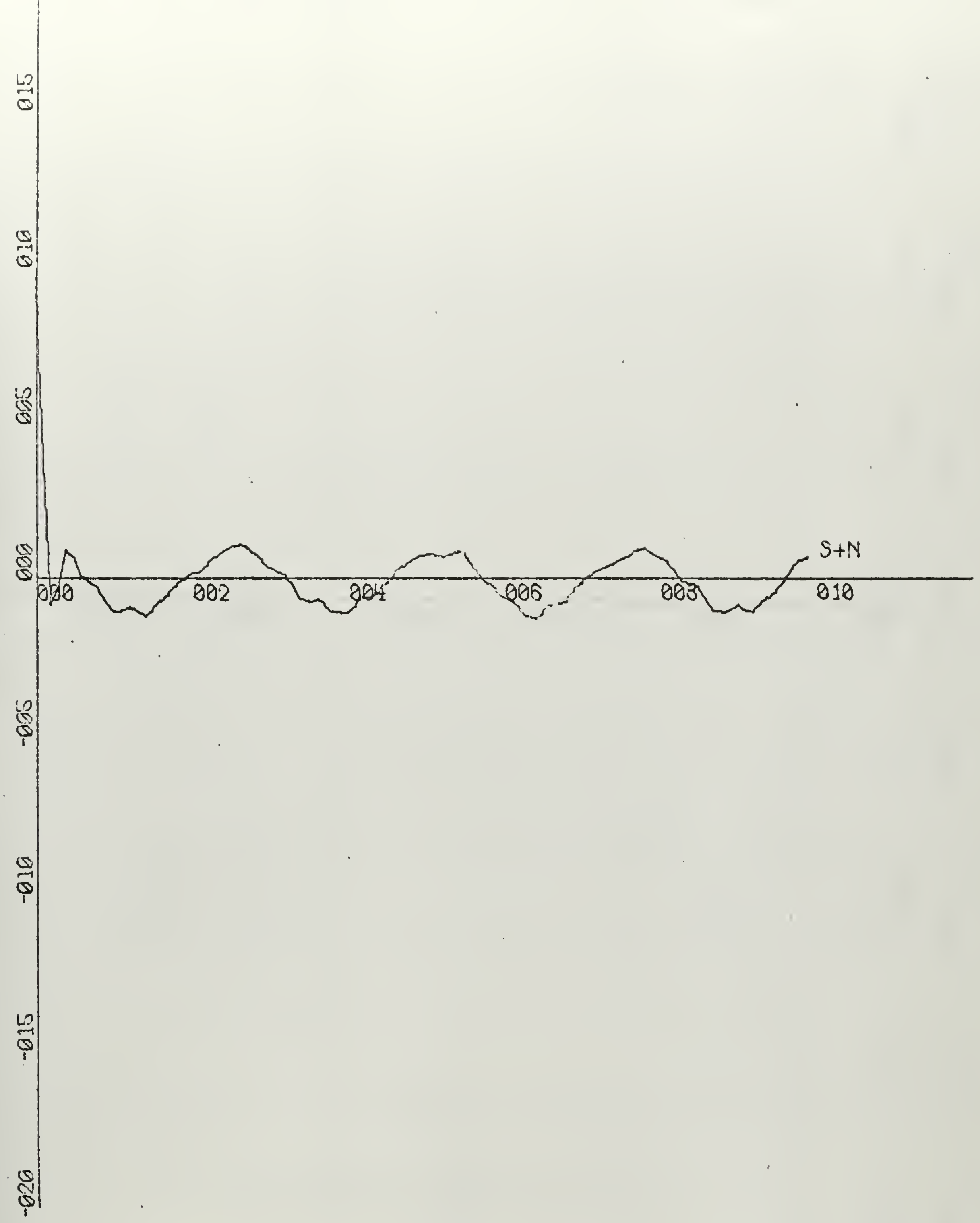
X Scale- .01 units/inch. Y Scale- 100 cps/inch.
Figure 14- -10DB. psd. after noise removal. 11250 samples.
Alpha determined on RBAR(T).



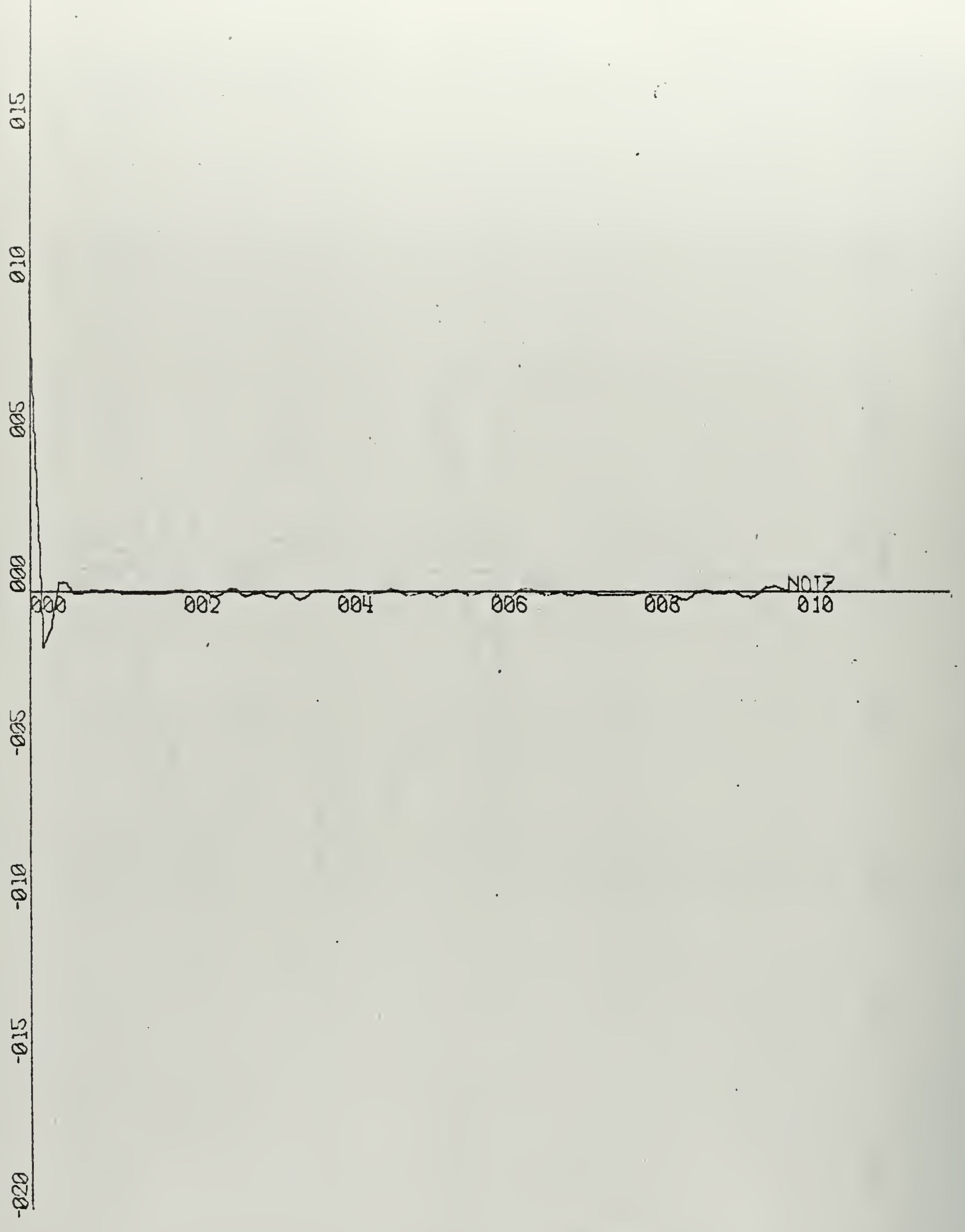
X Scale- .02 units/inch. Y Scale- 100 cps/inch.
Figure 15- -10DB. psd. after noise removal. 18750 samples.
Alpha determined on RBAR(T).



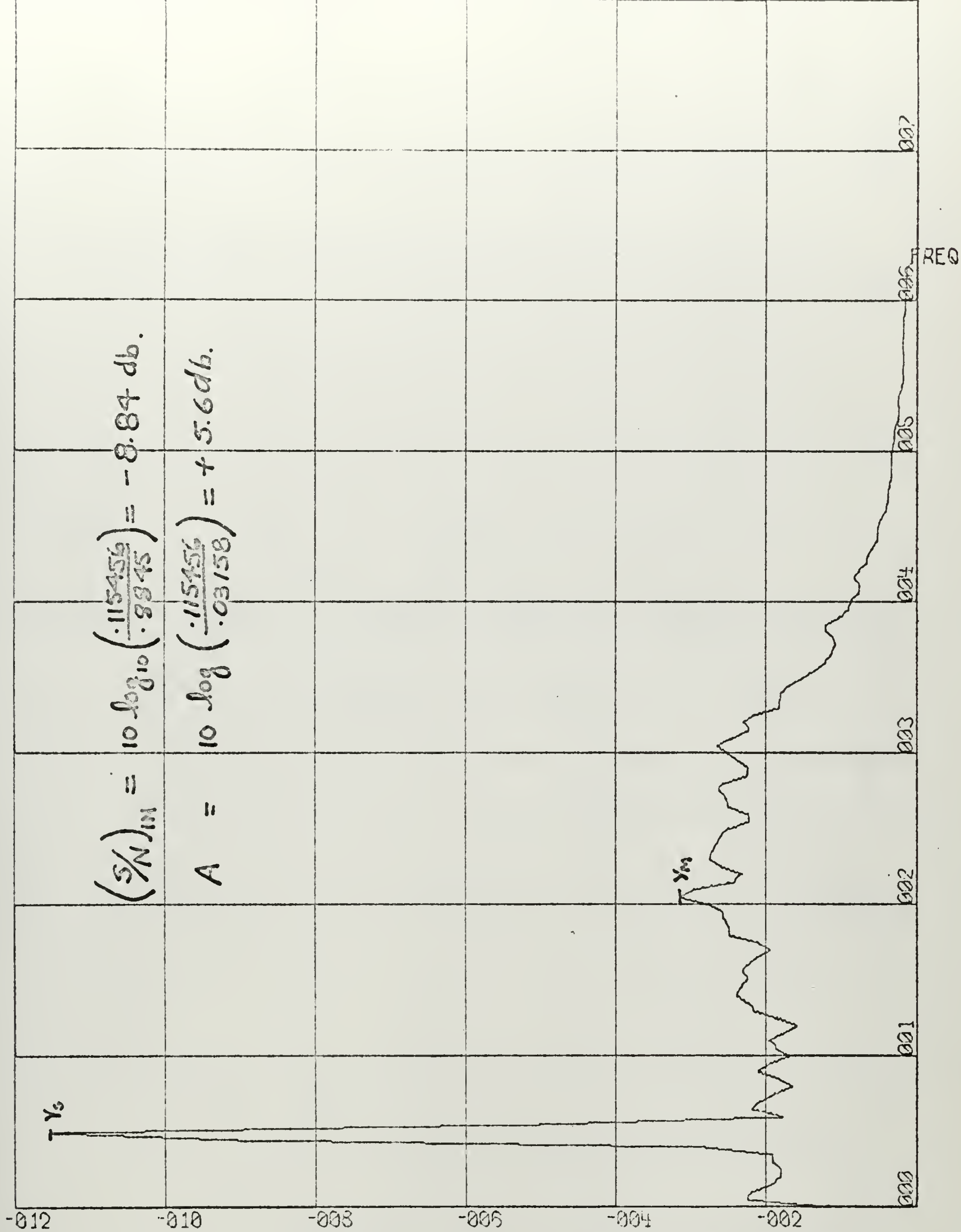
X Scale- 40 msec./inch. Y Scale- 0.5 units/inch.
Figure 16- -10DB. signal after noise removal. 18750 samples.
Alpha determined on RBAR(T).



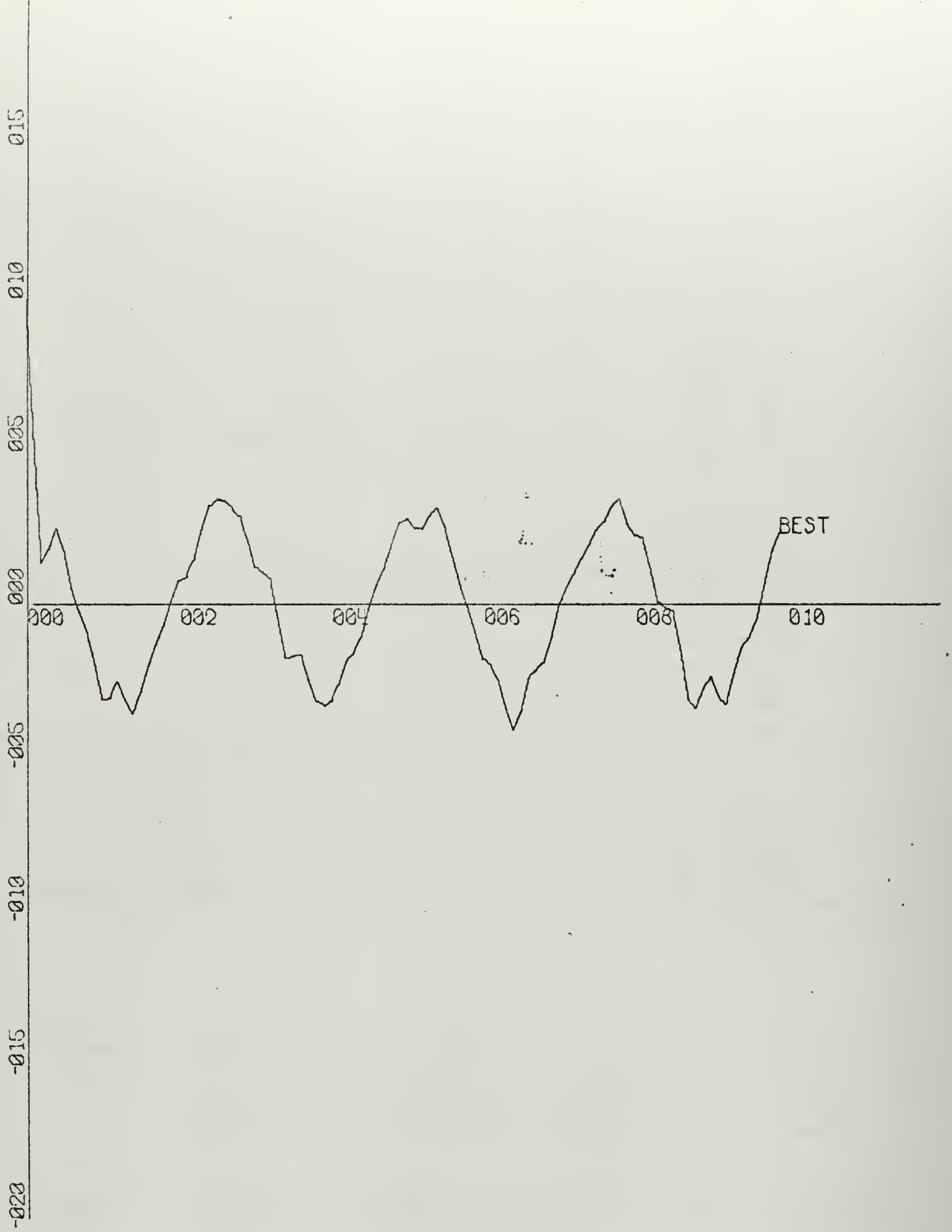
X Scale- 16 msec/inch. Y Scale- 0.5 units/inch.
Figure 17- -10DB. auto-correlation before noise removal. 22500 samples.



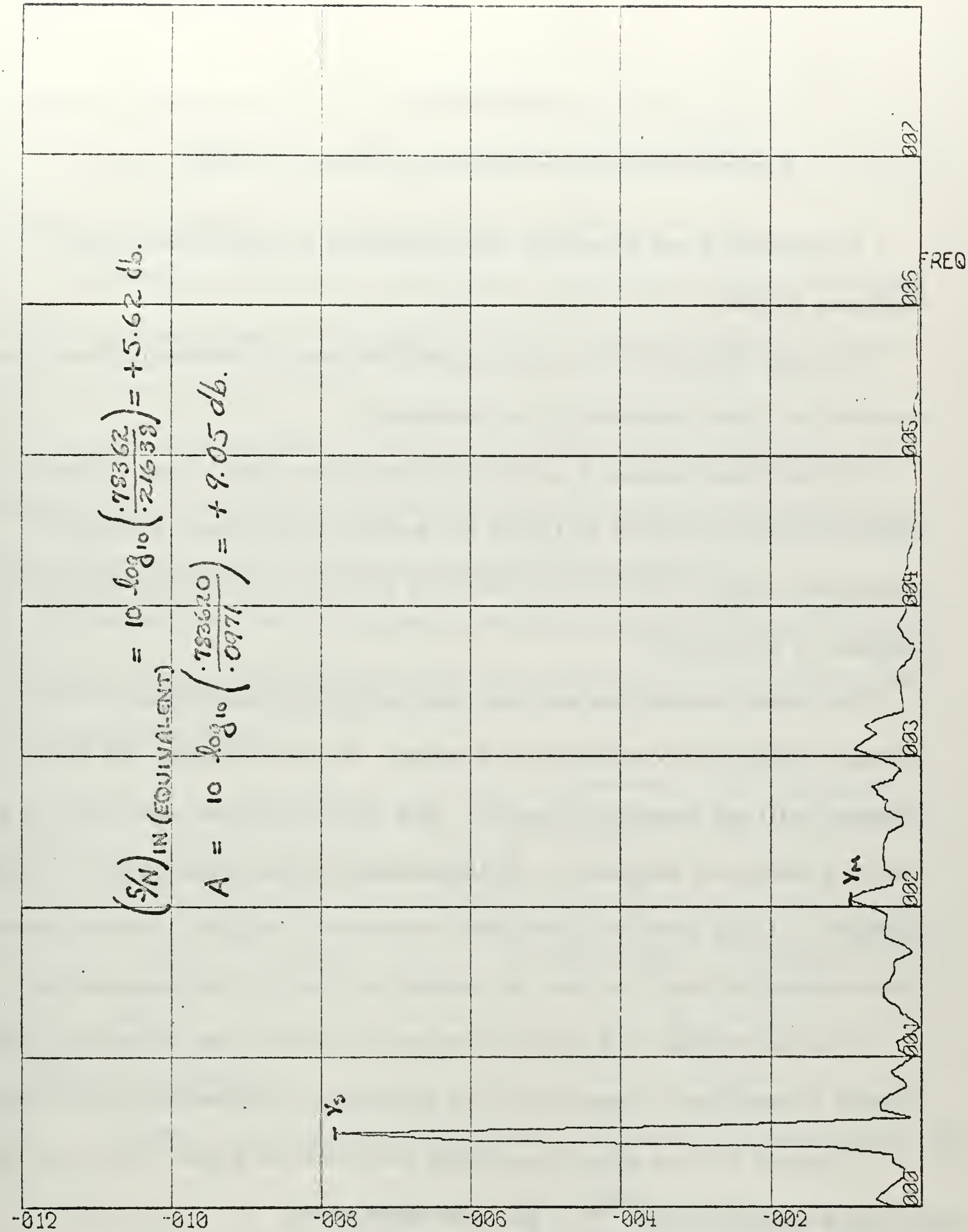
X Scale- 16 msec/inch. Y Scale- 0.5 units/inch.
Figure 18- Estimated auto-correlation of noise in -10DB. signal plus noise input, averaged over 37500 samples.



X Scale- .002 units/inch. Y Scale- 100 cps/inch.
Figure 19- -10DB. power spectral density before noise removal. 22500 samples, 125 estimates, 5 ops apart.



X Scale = 16 msec/inch. Y Scale - .5units/inch.
Figure 20 - -10 DB. SNR Signal after noise removal in frequency domain. 22500 samples.



X Scale = .02 units/inch. Y Scale - 100 cps/inch.

Figure 21 - Power spectral density of -10 DB. SNR after noise removal in frequency domain. 22500 samples.

APPENDIX C

DETECTION AND LOCATION OF SIGNALS IN NOISE

In Appendix A and B methods were developed for digital analysis of analogue signals.

The data utilized in the above appendices was in a sense artificial, in as much as it was generated in the laboratory.

To test the proceedures developed using a live signal, where signal characteristics and noise statistics are generally not known, a 200 cps sinusoidal signal source was placed in an unknown noise field produced by patrons of a cafeteria.

The signal source was set up at one end of the dining room, and its' volumn raised until complaints were voiced. The source level was then lowered until the complaints ceased. This level, which we shall refer to as the 0 db level was measured to be 38 milliwatts at the voice coil of the loudspeaker. At this level the sound was occasionally audible at various locations within the room, but was not detectable aurally at the microphones.

A stereo recorder was set up at the far end of the room, using two microphones symmetrically placed about the longtitudinal center line of the room.

A number of three minute recordings were made of signal plus noise, reduced signal plus noise (-9.3 db), and noise alone.

The noise level of the crowd varied very considerably over the hour and was punctuated by an occasional dropping plate. The period from 12:30 to 12:40 was selected for further study as the number of diners was essentially

constant over this period. The details are listed in Table I.

By cross-correlating the left and right channel information it is possible to locate the source and to estimate its frequency.

The bearing of a source, with respect to the center-line may be determined by measurement of the difference in arrival times of sound waves at the receiving sensors.

Assume as in Figure 1, a distant sinusoidal source, at an angle θ measured from the center-line or dead-ahead position, and two sensors symmetrically placed about the center-line spaced d units apart.

Assuming plane wave propagation, with no multipath effects, and in the absence of noise, the difference in travel time or phase of ray 2 is a direct measure of the angle θ.

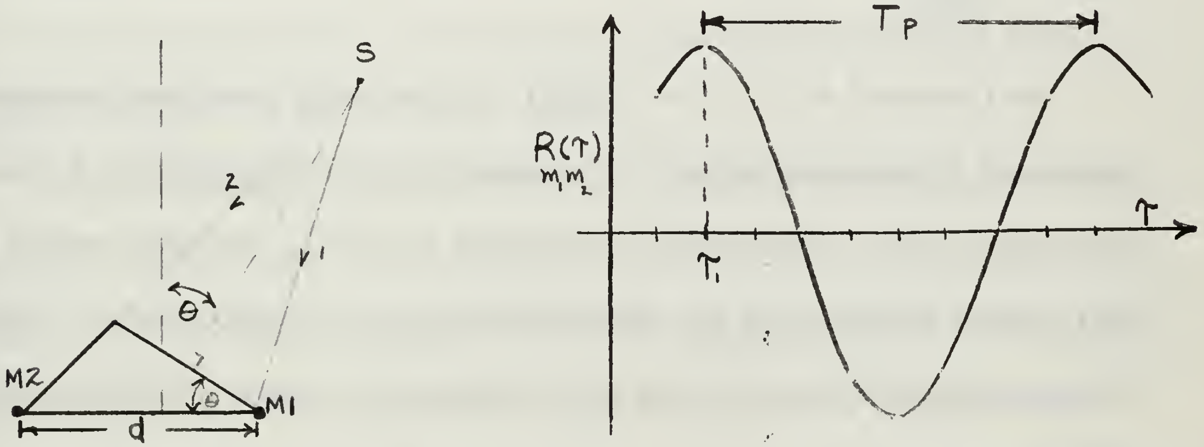


Figure 1. (a) The basic DF Problem. (b) Cross-Correlation of M_1, M_2.

Forming $$R_{M_1, M_2}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} (\cos w_o t)(\cos w_o(t-\tau))\,dt \qquad (1)$$

$$R_{M1M2}(\tau) = \frac{W_o}{2\pi} \int_{-\pi/W_o}^{\pi/W_o} \frac{1}{2}\left[\cos (W_o 2t + W_o\tau + (\gamma) \right] dt$$

$$+ \frac{W_o}{2\pi} \int_{-\pi/W_o}^{\pi/W_o} \cos\left[W_o \tau - \gamma\right] dt \qquad (2)$$

$$R_{M1M2}(\tau) = 0 + \cos (W_o\tau - \gamma) \qquad (3)$$

The first peak in the correlation curve occurs when

$$W_o \tau_1 = \gamma \quad \text{or} \quad \tau_1 = \frac{\gamma}{W_o} = \frac{d}{c} \sin \theta \qquad (4)$$

Subsequent peaks in $R_{M1M2}(\tau)$ occur when

$$W_o \tau - \gamma = N (2\pi), \quad N = 0, 1, 2, 3 \ldots$$

Peaks obtained for $n \neq 0$ represent ambiguities in the bearing angle. The first ambiguity occurs when $\tau_p = \frac{d}{c} \sin \Delta\theta$ or $\Delta\theta = \theta_a - \theta = \sin^{-1} \frac{CT_p}{d}$

$$= \sin^{-1} \frac{C\, 2\pi}{d\, W_o}$$

The first peak at τ_1 is of interest. In this case using room measurements and an assumed velocity of propogation of 1129 ft/second, τ_1 was calculated to be 1.5506 msec. for an angle of 10.4°. For small angles we may assume the time and the associated angle are linearly related. Under this assumption the arrival time would change very nearly 150 μ sec. per degree.

The available processing system however could sample the record at best every 200 μ sec., resulting in a bearing accuracy of about 1.33°. To improve the bearing accuracy, the data was slowed by a factor of four, grequency shifting all components down by a factor of four. This has the

effect of increasing the arrival time by a factor of four resulting in a bearing accuracy of $1.33/4^{\circ}$ or $.333^{\circ}$.

The mechanics of the speed reduction are shown in Table 2.

This is more bearing accuracy than is justifiable but since a reduction in data speed of at least two was necessary to digitize a 5 Kc analogue record, the extra factor of two was used to increase bearing accuracy.

The data speed reduction meant that the original 200 cps signal would appear as 50 cps with the first peak of the cross-correlation curve at τ = 6.02 msec. The data was sampled at intervals of 205 μsec., and hence the signal, if present, should occur between 29 and 30 shifts of the correlation function argument.

2. Data Analysis

Twenty blocks of data were digitized for each of three cases; signal to noise ratio 0 db; -10 db; and noise alone.

Due to difficulties experienced with the clocking system, only the first block of a twenty block run was clocked, with clocking of subsequent blocks internally. Due to the variations in the time necessary for the 163 tape unit to recover from a writing operation, a cumulative error in clocking arose: Hence only the first few blocks are time synchronized to an acceptable degree. Fortunately this was long enough for sucessful signal detection.

The data was normalized in the computer by the computed estimator of the standard deveation, and the cross-correlation of the normalized data formed.

The time of integration was fixed due to the finite block length. Assum-

ing a process is stationary and ergodic, (at least short-time stationary) the effective integration time may be extended by averaging in time, ie

if $$R_i ab(\tau) = \frac{1}{N} \sum_{N=1}^{N} f_a(t) f_b(t-\tau) \tag{5}$$

and $$\bar{R}_i ab(\tau) = \frac{1}{K} \sum_{i=1}^{K} R_i ab(\tau)$$

$$\bar{R}_i ab(\tau) = \frac{1}{K} \sum_{i=1}^{K} \left(\frac{1}{N} \sum_{N=1}^{N} f_a(\tau) f_b(t-\tau)\right)$$

$$= \frac{1}{NK} \sum_{1}^{NK} f_a(\tau) f_b(t-\tau) \tag{6}$$

The running average cross-correlation function of the 0 db signal plus noise was formed over the first five blocks and is shown in Figures 3 through 5 as a "hatched-in" curve.

3. Results

Experiments with averaging more than five curves were not successful, due to the previously mentioned cumulative timing error.

The cross-correlation of 0 db signal plus noise was observed to be essentially sinusoidal with the first peak occuring between 29 and 30 shifts exactly as predicted.

The frequency of the sinusoid however was observed to be 150 cps, or three times the expected signal frequency, indicating detection was being made on the third harmonic of the source!

No firm explanation for this is offered. However it is noted that the

oscillator produced 0.1% third hormonic, and the 20 W. push-pull amplifier used would be expected to produce considerable cross-over distorion when operated at such low power levels. In addition the tape re-recording process tended to favour the upper registers.

A power spectral density was made of the 0 db signal plus noise, and of noise alone, using a 6.125 cps filter bandwidth. The normalized results, shown in Figure 2, reveal the pronounced 150 cps component in the signal recording, with the general shape of the remaining portion being similar to the noise power spectral density.

The programming mecessary is included as Program SIMSIG in Appendix B.

The results obtained with the signal at -9.3 db were inconclusive and are not included.

4. <u>Conclusion</u>

Provided accurate time synchronization between right and left channels can be maintained during the digitizing process, cross-correlation of the two channels provides both bearing and frequency information.

The process of averaging cross-correlation curves must be done with care, ensuring accurate time synchronization and continuing the averaging process only as long as the signal characteristics remain stable.

ANALOGUE TIME	DATA STATE	DIGITAL RUN	
12:30-12:33	"O" DB.	F	0001 0021 0001 0024 (R) TO 0001 0021 0024 0024
			0001 0032 0001 0024 (L) TO 0001 0032 0024 0024
12:35-12:33	"-10"DB.	J	0002 0021 0001 0024 (R) TO 0002 0021 0024 0024
			0002 0032 0001 0024 (L) TO 0002 0032 0024 0024
12:41-12:44	NOISE	C	0003 0021 0001 0024 (R) TO 0003 0021 0024 0024
Digitized on Reserve Tape 18 Sampling Interval 205 μs.			0003 0032 0001 0024 (L) TO 0003 0032 0024 0024

TABLE I: Details of Data Digitized.

STEP	PLAYBACK	BAND-W.	RECORD	BAND-W.	SIGNAL
	Concertone 7.5 IPS	50 cps 10 Kc.	Ampex FR-100	0 - 1.1 Kc.	200 cps
	Ampex 7.5 IPS	0 - 650 cps	Concertone 7.5 IPS	20 - 650 cps	100 cps
	Concertone 3.75 IPS	10 - 325 cps	Ampex 7.5 IPS	10 - 325 cps	50 cps

TABLE II: Frequency Reduction Process.

```
..JOB0148F,BARRETT,N.A.
      PROGRAM LOCATE
  100 FORMAT(15H THETA DEGREES )
  101 FORMAT(F10.7)
  102 FORMAT(14H ZETA DEGREES )
  103 FORMAT(F10.7)
  104 FORMAT(13H RHO DEGREES )
  105 FORMAT(F10.7)
  106 FORMAT(15H ALPHA DEGREES )
  107 FORMAT(F10.7)
  108 FORMAT(12H SI DEGREES )
  109 FORMAT(F10.7)
  110 FORMAT(16H DELAY IN MSECS )
  111 FORMAT(F10.7)
  112 FORMAT(1H1)
      DEGRAD= 57.2957795
      TANTHET=13.0/70.75
      THETRAD=ATANF(TANTHET)
      THETDEG=THETRAD*DEGRAD
      TANZET=8.166667/70.75
      ZETRAD=ATANF(TANZET)
      ZETDEG=ZETRAD*DEGRAD
      RHODEG=90.0-ZETDEG
      ALPHDEG=180.0-(RHODEG+THETDEG)
      TANBTTH=70.75/17.83333
      BTTHRAD=ATANF(TANBTTH)
      BTTHDEG=BTTHRAD*DEGRAD
      BTTWDEG=90.0-ALPHDEG
      BTAODEG= 90.0 - BTTHDEG - BTTWDEG - ZETDEG
      GAMDEG=90.0-BTAODEG
      SIDEG=180.0-GAMDEG
      SIRAD=SIDEG/DEGRAD
      TAU=SINF(THETRAD)*9.6666667/(SINF(SIRAD)*1.129)
C     TAU IN MILLISECS
      PRINT 112
```

```
PRINT 100
PRINT 101,THETDEG
PRINT 102
PRINT 103,ZETDEG
PRINT 104
PRINT 105,RHODEG
PRINT 106
PRINT 107,ALPHDEG
PRINT 108
PRINT 109, SIDEG
PRINT 110
PRINT 111,TAU
PRINT 112
END
END
```

Computed Results

$\theta = 10.411^\circ$
$\xi = 6.584^\circ$
$\rho = 83.4154^\circ$
$\alpha = 86.173^\circ$
$\varphi = 93.7356^\circ$
$\Delta t = 1.5506$ msec.

Velocity of Sound taken to be 1129'/sec

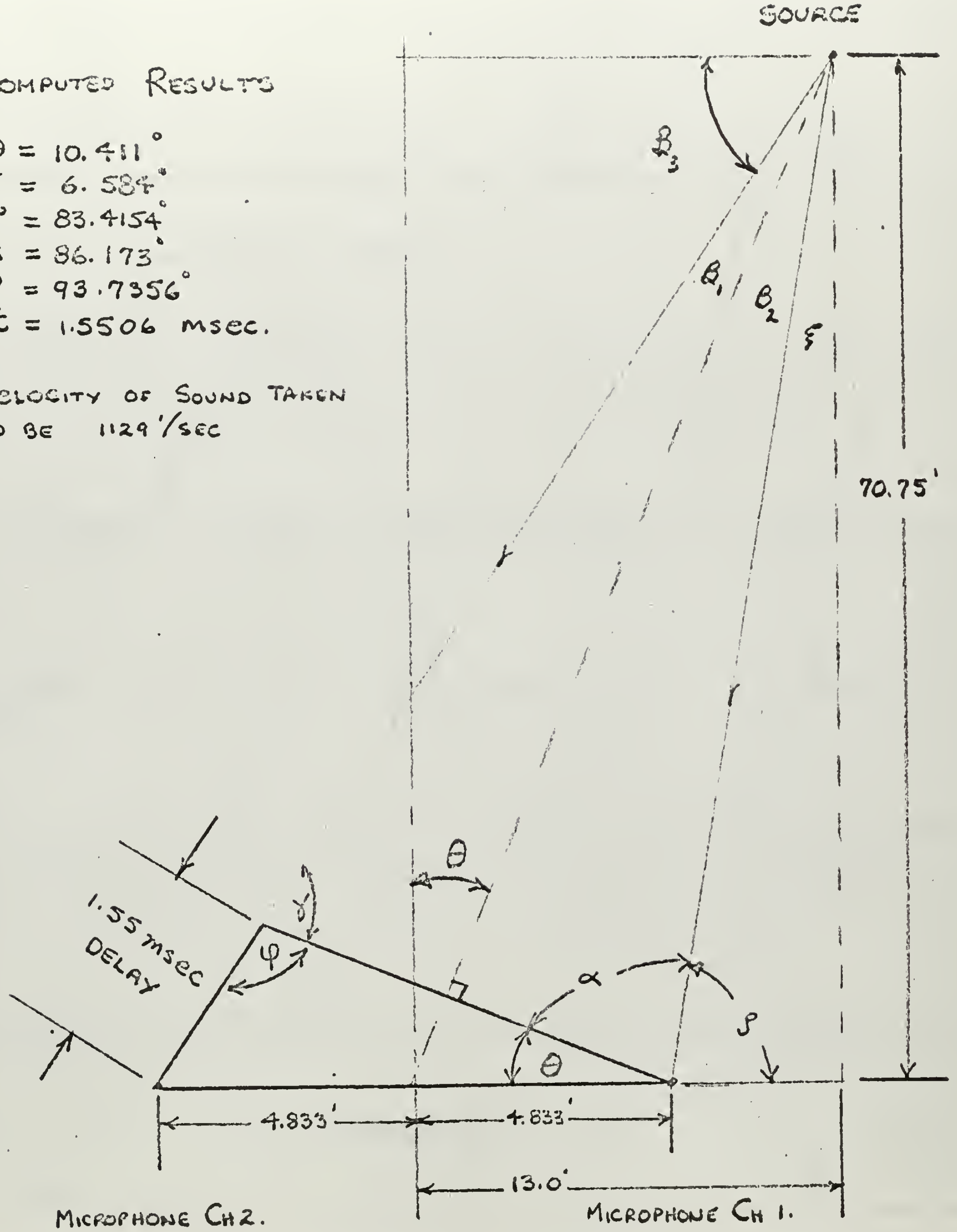
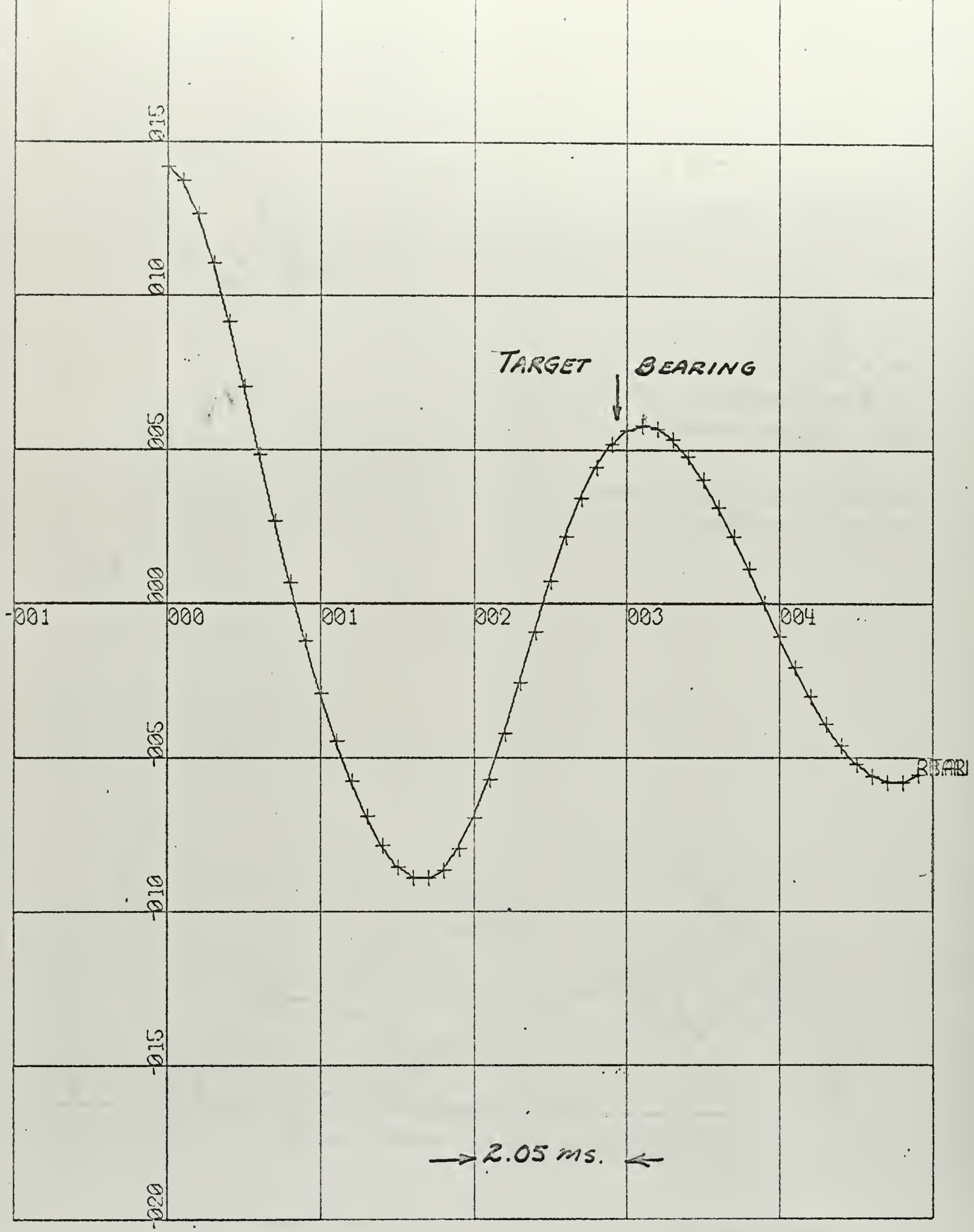
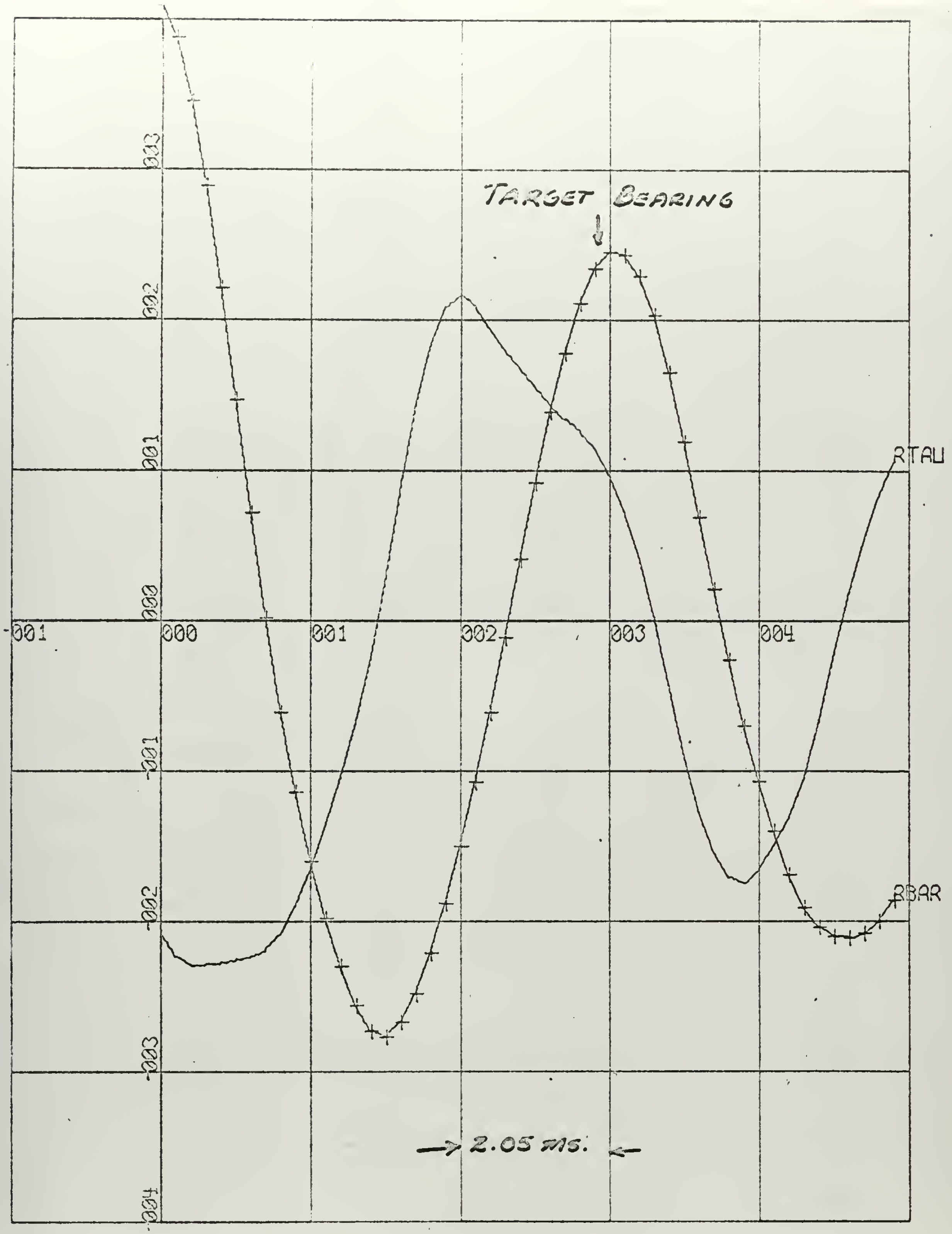


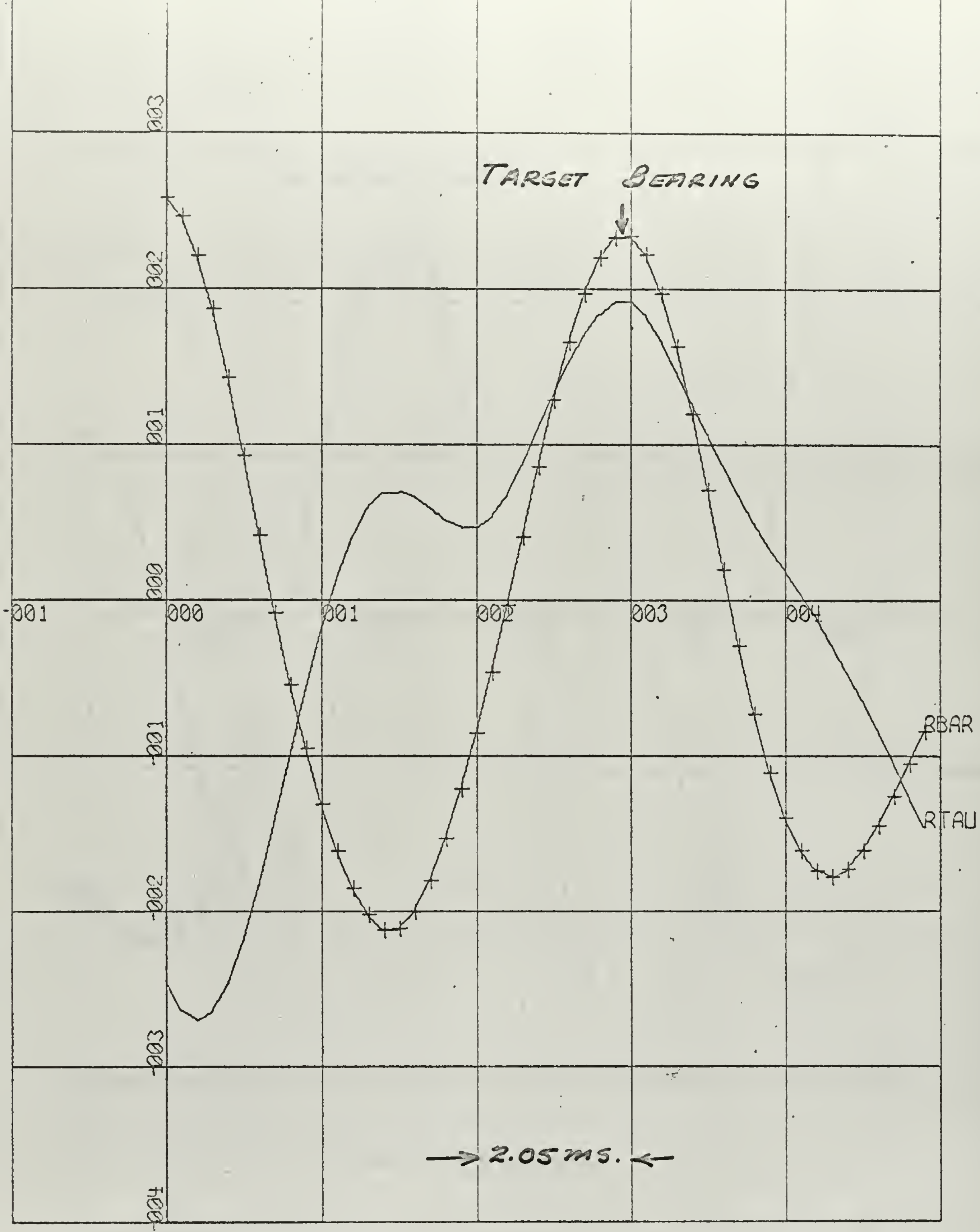
Figure 2: Geometry of the Lunch-Room problem



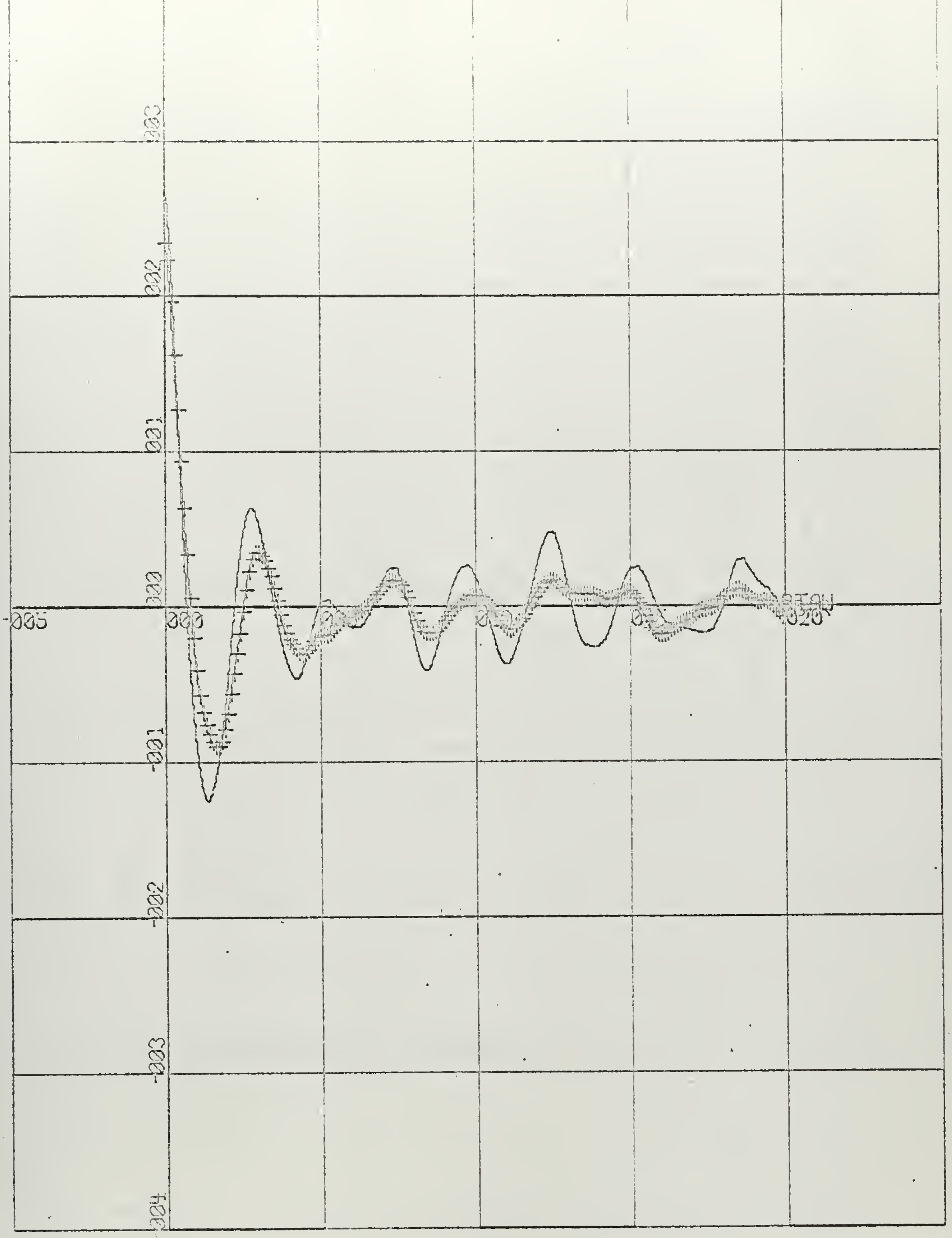
X Scale- 2.05 msec/inch. Y Scale- 0.5 units/inch.
Figure 3- ODB. signal cross-correlation over 3950 samples.



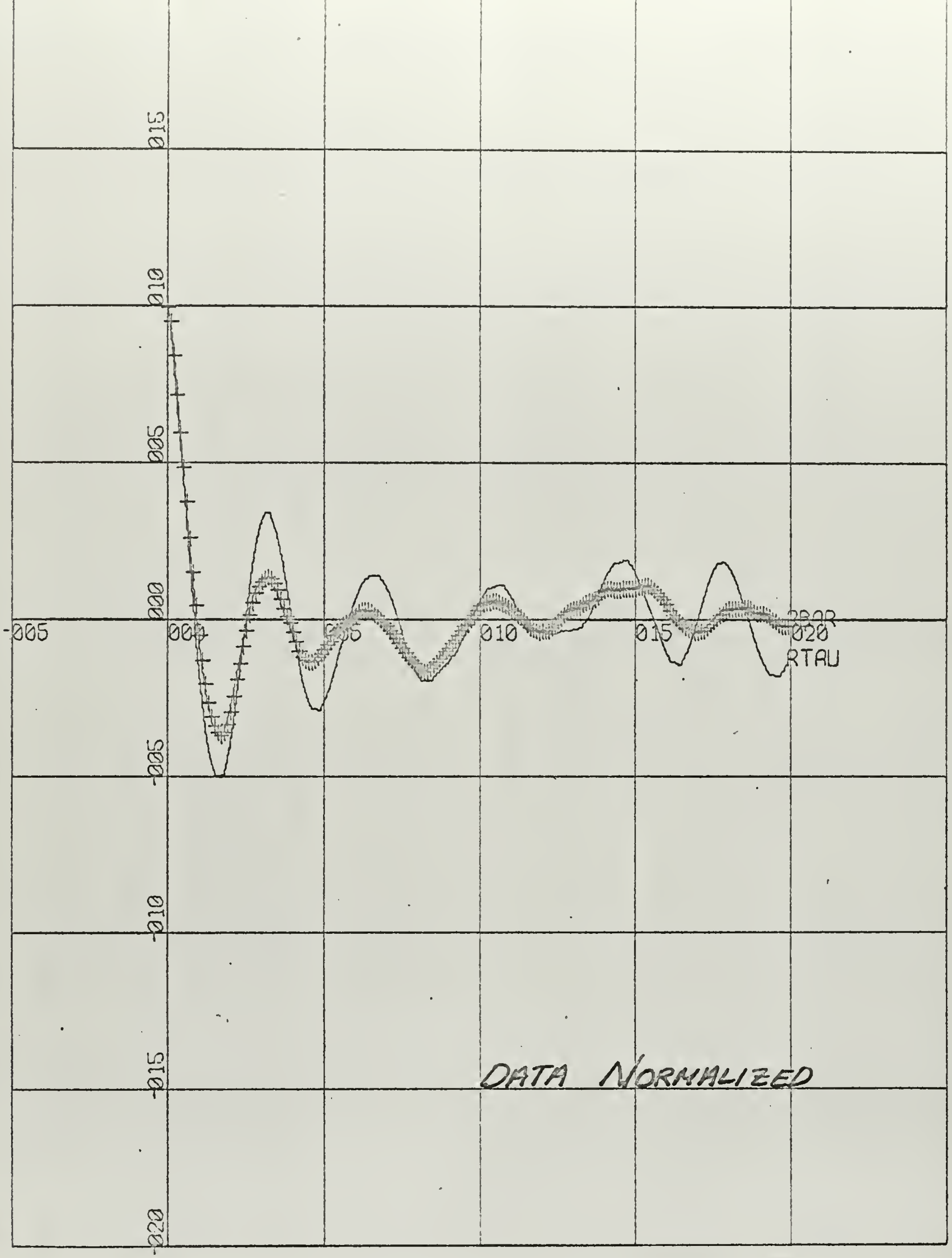
X Scale- 2.05 msec/inch. Y Scale- 0.5 units/inch.
Figure 4- ODB. signal cross-correlation over 11850 samples.



X Scale- 2.05 msec/inch. Y Scale- 0.5 units/inch.
Figure 5- ODB. signal cross-correlation over 19750 samples.



X Scale- 10.25 msec/inch.   Y Scale- 0.1 volts^2/inch,
Figure 6- 0DB. signal auto-correlation over 11850 samples.



X Scale- 10.25 msec/inch. Y Scale- 0.5 volts^2/inch.
Figure 7- Noise auto-correlation over 11850 samples.

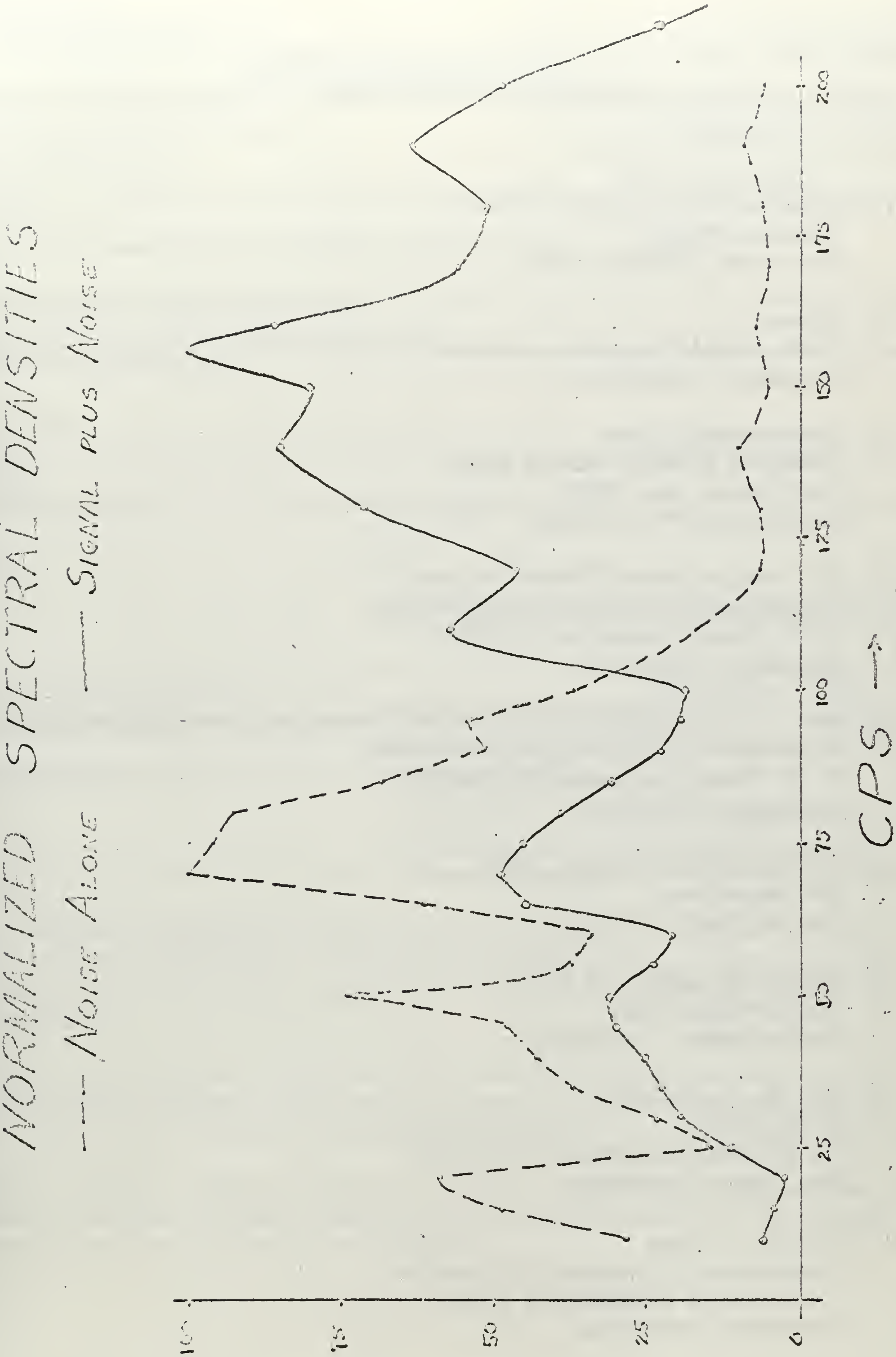


Figure 8- Relative power spectral densities of noise alone, and signal plus noise. Normalized to 100% .

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1. ORIGINATING ACTIVITY *(Corporate author)* U. S. Naval Postgraduate School Monterey, California	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b. GROUP NOT APPLICABLE

3. REPORT TITLE

Extracting Analogue Signals from Noise Using a Digital Computer

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

Master's Thesis in Engineering Electronics

5. AUTHOR(S) *(Last name, first name, initial)*

Barrett, Neil A.

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
May, 1966	144	7

8a. CONTRACT OR GRANT NO. b. PROJECT NO. c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) 9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*

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13. ABSTRACT

It is frequently convenient in data processing to convert analogue to digital data for computer assimilation. A convenient method of such conversion has been developed and used in the study of correlation detection of the audio signals corrupted by noise.

A method to use apriori knowledge of the corrupting noise to increase processing gain has been studied. In the case of detection of a sinusoid in noise, an additional gain over convertial auto-correlation of up to 14.5 db has been achieved.

Finally, a signal source located in an unknown random noise field was detected classified and located in relative bearing by the cross-correlation of the signals received from two spatially separated sensors.

DD FORM 1 JAN 64 1473

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14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
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Correlation Detection						
Noise Removal						

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